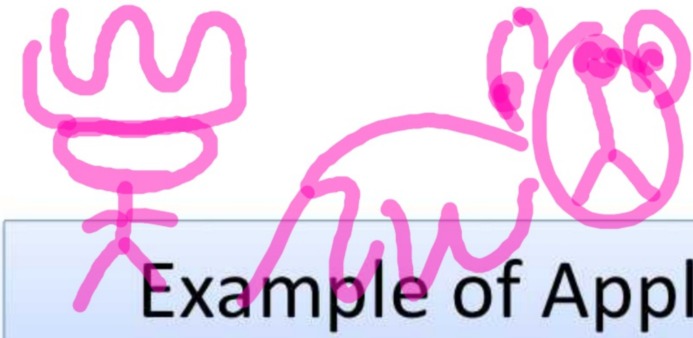


Example of Applications of Quadratics

C = Cost
R = Revenue

Profit = R − C

Areas of Squares and Rectangles

Path of an object when thrown in the air

By: Fares Sammir

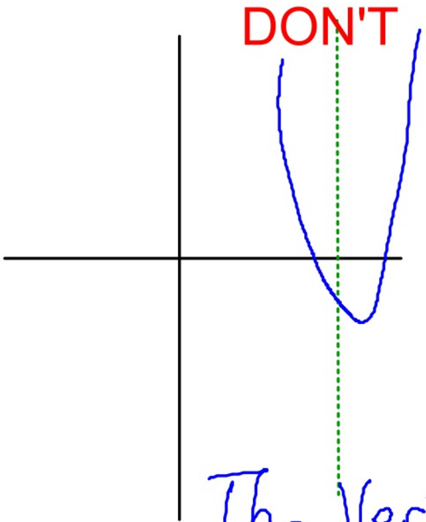
Remember when graphing!

axis

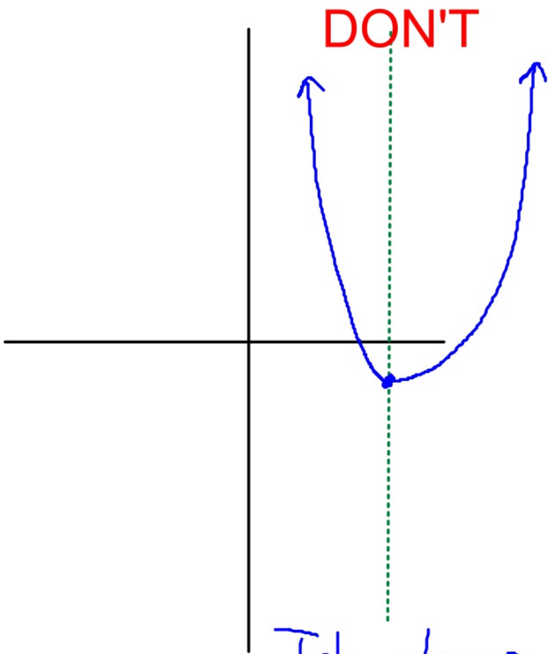
- Label your axis. If it is a "real life" example, then label accordingly. Do not use x and y.
- Independent variable is always graphed on the horizontal axis (x-axis)
- Dependent Variable is always graphed on the vertical axis (y-axis)
- Follow instructions for the scale of both axis. If instructions are not given, think of your values before you decide on an appropriate scale.
- Check for limits in your Domain and Range. You only graph for the limited domain.
- Mark the AoS as a "dashed-lined" (tac tac tac) and plot the Vertex.
- Draw enough points to draw a smooth curve. (table of smart points). Parabolas are always symmetrical, with the Vertex on the AoS.

you can use the table in your GDC for your smart points!

I don't want to see the following sketches!

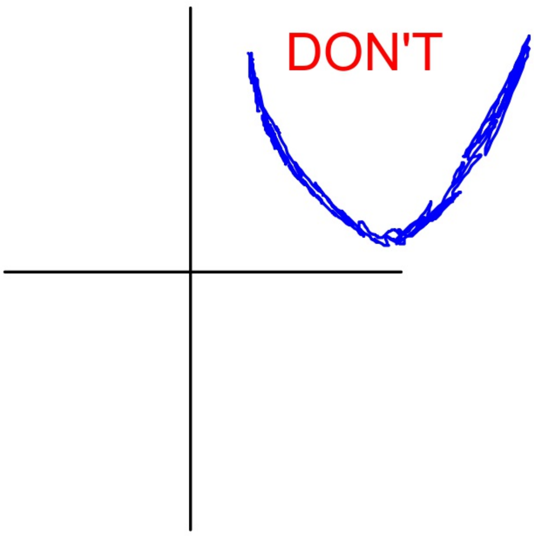


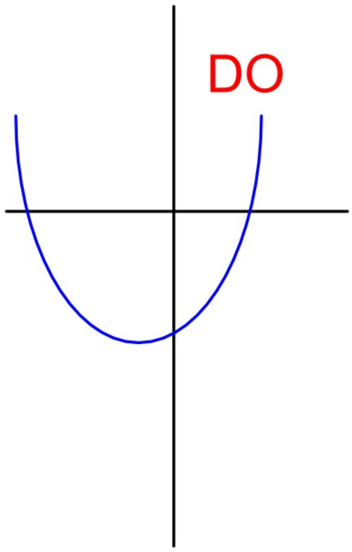
The Vertex does Not sit on the AoS and it should



It does NOT look Symmetrical

I don't want to see "free hand" sketching





Typical example: Area of rectangle

- A rectangle has a perimeter of 36 cm. If we let x represent the width of the rectangle:

i. Find the possible values that x can take.

ii. Find the length of the rectangle in terms of x

iii. Show that the area A(x)=x(18-x)

iv. Sketch the graph of A against x

v. Find the dimensions of the rectangle which has the largest possible area for the given perimeter.

i) Find the possible values that x can take?

- Consider that x represents a length

x represents a length so it cannot be 0 or less, and because the perimeter is 36, then x has to be smaller than half the perimeter, so it as to be less than 18 and not equal to 18

$$\therefore 0 < x < 18$$

Note: $x \neq 0$ and $x \neq 18$

so, if you have to graph, your domain would be between 0 and 18. This information should help you scale your axis

ii) Find the length of the rectangle in terms of x

- Use the perimeter for this.

For any rectangle: $P = 2w + 2\ell$

For our rectangle: $36 = 2x + 2\ell$

Now we want to solve for the length in terms of x:

$$\frac{36 - 2x}{2} = \frac{2\ell}{2}$$

$$\boxed{\ell = 18 - x}$$

iii) Show that the area A of the rectangle is given by A(x)=x(18-x)

- Remember what "show that" stands for?

For any rectangle: $A = w \cdot \ell$

For our rectangle: $A = x(18 - x)$

Hence, area is now a quadratic function depending on the width "x"

$$A(x) = x(18 - x)$$

or

$$A(x) = 18x - x^2$$

iv) Sketch the graph of Area against x

It is a sketch, so no need to be so exact. However, you still need to:

- ☑ Label your axis
- ☑ Maintain a "sense of a scale"
- ~~☐ Plot the plots~~

$A(x) = x(18-x)$

$A(x) = 18x - x^2$

$b=18 \quad a=-1$

Using standard form

$x = \frac{-b}{2a}$

$x = \frac{-18}{2(-1)} = 9$

using factor form $x = \frac{18+0}{2}$

Note: you can use the standard form to find AoS and Vertex. $x=9$
or you may use the Farctor form

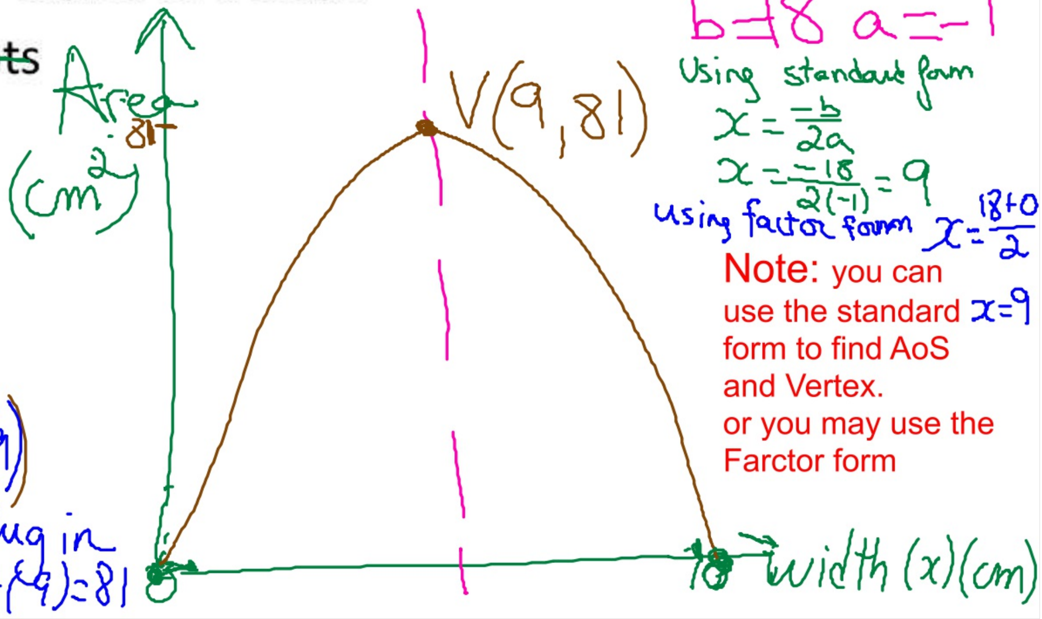
AoS

$x=9$

$V(9, A(9))$

plug in

$A(9)=81$



v) Find the dimensions of the rectangle which has the largest possible area

- Which point would correspond to this?

At the VERTEX

Width = 9cm

length = 18 − 9

= 9cm

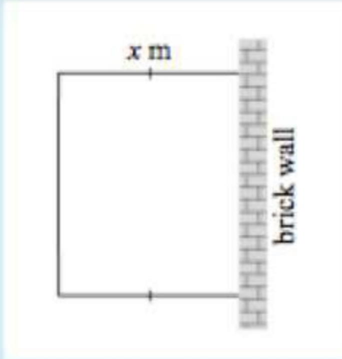
$\boxed{9\text{cm by } 9\text{cm}}$

Example 35

A vegetable gardener has 40 m of fencing to enclose a rectangular garden plot where one side is an existing brick wall. If the two equal sides are x m long:

a show that the area enclosed is given by $A = x(40 - 2x)$ m^2

b find the dimensions of the vegetable garden of maximum area.



$P = 2x + l$

$40 = 2x + l$

$40 - 2x = l$

$A = W \times L$

$A = x(40 - 2x)$

$A = 40x - 2x^2$

$AOS = \frac{-40}{-4}$

$x = 10$

Maximum at

$V = (10, ?)$

$x = 10 \quad l = 20$

Example 1

YOUR TURN TO TRY!

Example 36

A manufacturer of pot-belly stoves has the following situation to consider.

If x are made per week, each one will cost $\left(50 + \frac{400}{x}\right)$ dollars and the total receipts per week for selling them would be $(550x - 2x^2)$ dollars.

How many pot-belly stoves should be made per week in order to maximise profits?

Cost function.

$Cost = 50 + \frac{400}{x}$

$\therefore C(x) = x\left(50 + \frac{400}{x}\right)$

$C(x) = 50x + 400$ Linear!!!!

This part was quite tricky. In Math Studies we do not deal with reciprocal functions, which is (50 + 400/x) However, when you multiply by each pot-belly stove (x), then you end up with a linear function

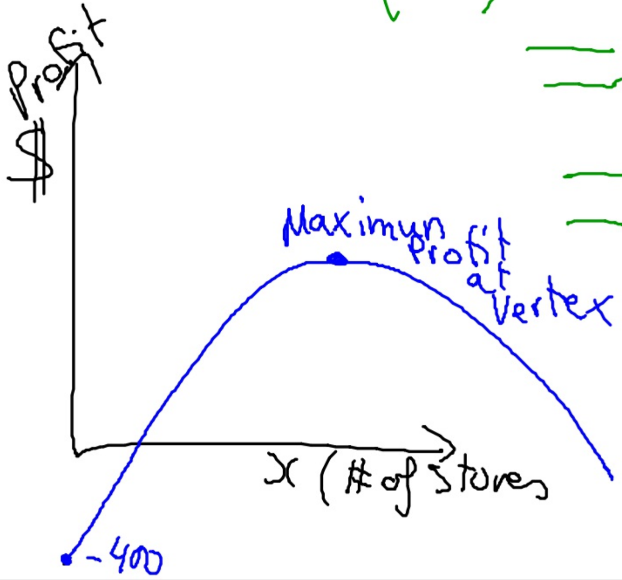
$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P(x) = (550x - 2x^2) - (50x + 400)$$

$$P(x) = 550x - 2x^2 - 50x - 400$$

$$= 500x - 2x^2 - 400$$

$$= -2x^2 + 500x - 400$$



Only answer what they want. The question is only asking for the number of stoves to obtain maximum profit. This means, they only need the x-coordinate of the vertex, not the y-coordinate which is the actual maximum profit.

$V(x, P)$

P: the max. profit

x: not the maximum stoves but the number of stoves to obtain max profit

$$x = \frac{-b}{2a}$$

$$x = \frac{-500}{2(-2)}$$

$$x = \frac{-500}{-4}$$

$$\boxed{x = 125}$$

When 125 stoves are sold, we have max. profit.

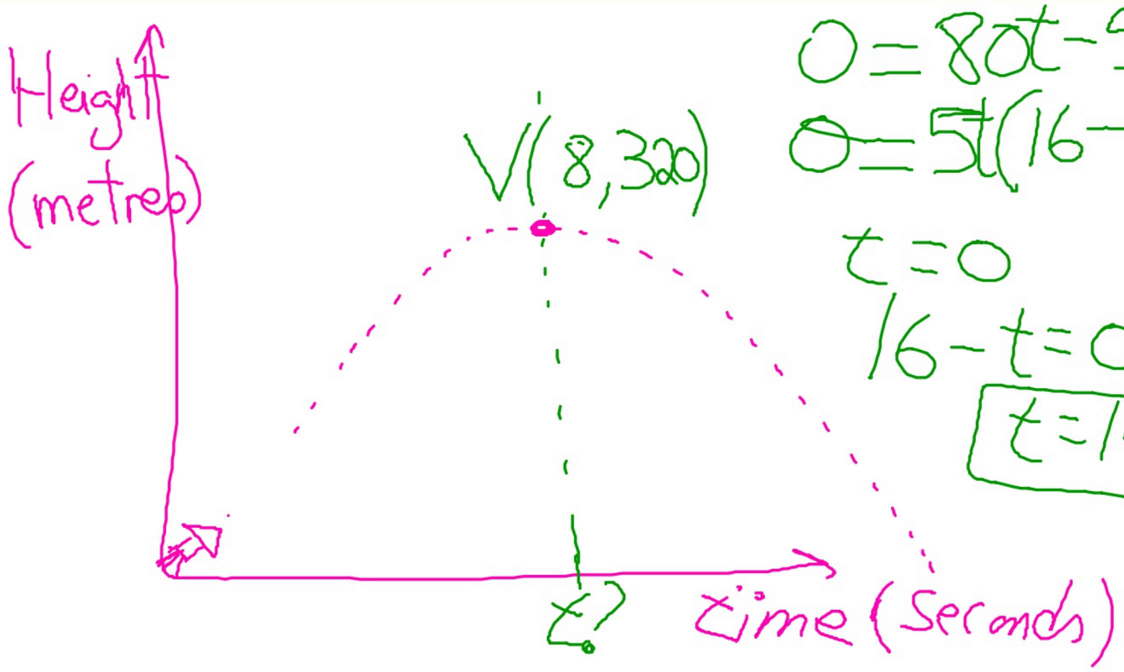
Example 2

The height H metres, of a rocket t seconds after it is fired vertically upwards is given by $H(t) = 80t - 5t^2$, $t \geqslant 0$.

a How long does it take for the rocket to reach its maximum height? 8 s

b What is the maximum height reached by the rocket? 320 m

c How long does it take for the rocket to fall back to earth? 16 s



$0 = 80t - 5t^2$

$0 = 5t(16 - t)$

$t = 0$

$16 - t = 0$

$\boxed{t = 16s}$

Ex 3. A firm's profit function, $P, is $P = -x^2 + 20x - 60$, where x represents the quantity of goods sold per day.

a) Draw the graph of the profit function for $0 \leqslant x \leqslant 25$, $0 \leqslant y \leqslant 50$.

b) Find the profit when i) 10 units, and ii) when 20 units are sold.

c) Determine the maximum profit for the firm and the quantity of goods sold to make that profit.

a) From your calculator get a table of smart points to graph this parabola.

x	y
0	-60
40	-60

not part of the range

x	y
5	15
15	15
8	36
12	36

b) You can know use the graph to find profit at 10 units and 20 units. Or you may use the equation.

$P(10) = -(10)^2 + 20(10) - 60$

$-100 + 200 - 60$

$P(10) = 40$

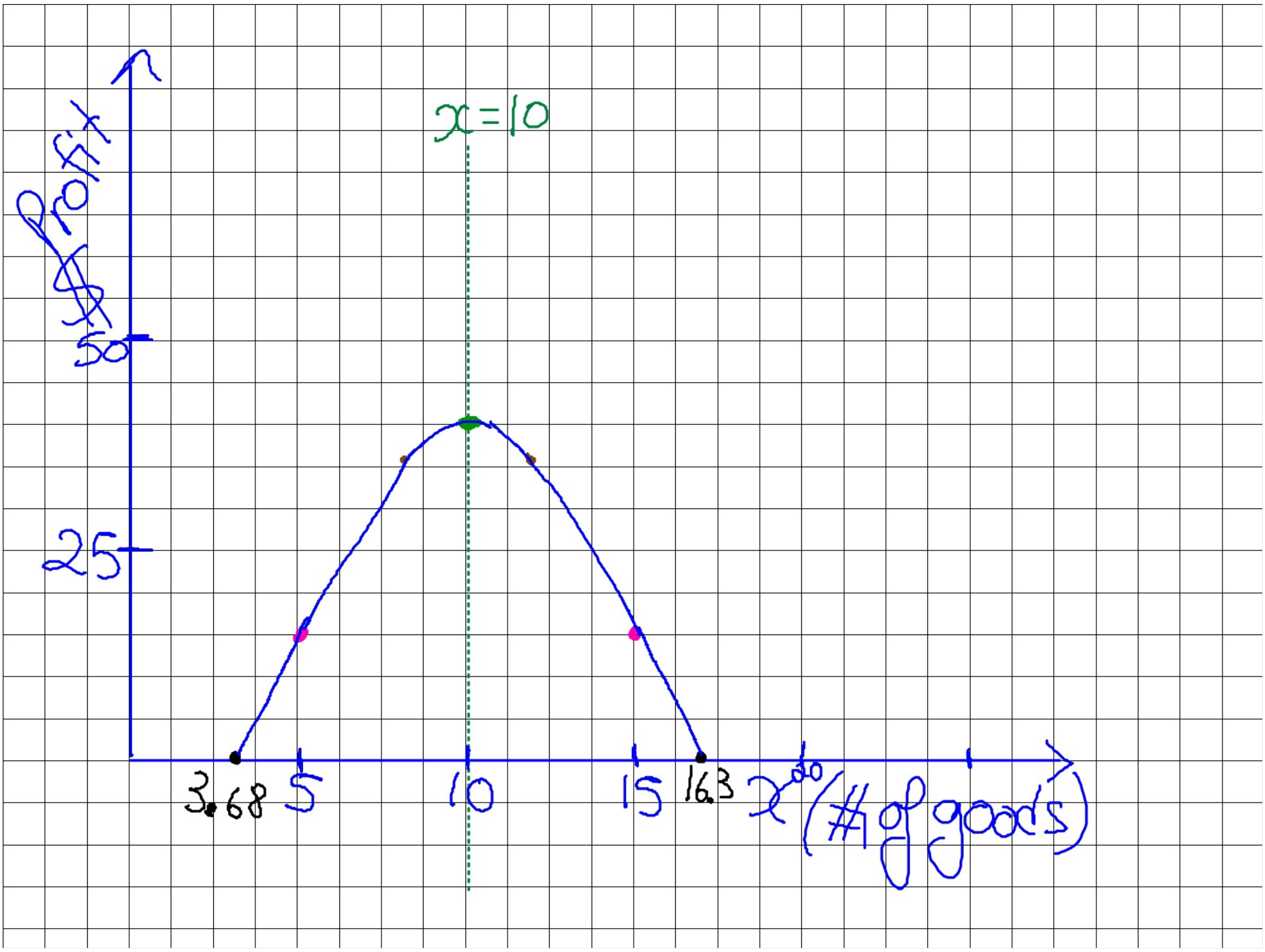
$P(20) = -(20)^2 + 20(20) - 60$

$P(20) = -60$

Note: when they sell 20 units, the firm looses money

c) Maximum profit will occur at the vertex. In this question, they are asking for both coordinates of the vertex.

$x = \frac{-b}{2a} \Rightarrow x = \frac{-20}{2(-1)} \Rightarrow x = 10$ At 10 units we have max profit $\therefore$ \$40 ($P(10)$) is max profit



Note: that the path of the ball is not actually a parabola, however the height function depending on time (independent variable) is indeed a quadratic function, hence graphed as a parabola.

Example 4

1 The height H metres, of a ball hit vertically upwards t seconds after it is hit is given by $H(t) = 36t - 2t^2$.

- **a** How long does it take for the ball to reach its maximum height?
- **b** What is the maximum height of the ball?
- **c** How long does it take for the ball to hit the ground?

a) The maximum height will be at the Vertex of the Parabola. Since the function is given in standard form, we have to use x=-b/2a. This will give us the time at the maximum height (x-coordinate of the vertex)

$$x = \frac{-36}{2(-2)}$$

$$x = 9$$

$$\therefore t = 9 \text{ seconds}$$

b) The actual maximum height is the y-coordinate of the vertex or the value of the function at t= 9 s

$$H(9) = 36(9) - 2(9)^2$$

$$H(9) = 162 \text{ m}$$

c) When it hits the ground H=0, so you must solve the equation $0=36t-2t^2$ for t.

You can solve this equation by factoring, it is not too hard: 0=2t(18-t) ===> t=0 or 18-t=0 ==> t=18

You can also solve using your GDC, polynomial tools==> a_2=-2, then a_1=36 and a_0=0 You also get t=0 or t=18

Hence, the answer is t=18 seconds. The answer t=0 does not apply here, bc it represents the height before the ball is actually hit vertically.

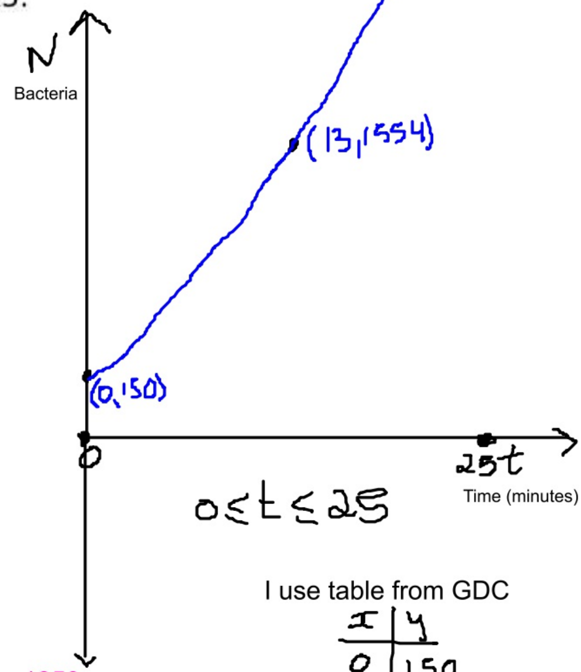
Ex. 5. The number of bacteria, N, in a culture t minutes after the start of an experiment is given by $N = 150 + 69t + 3t^2$.

a) determine the number of bacteria present at the start of the experiment.

b) Find the number of bacteria present after 5 minutes.

c) Find the time required for the number of bacteria to reach 2000.

d) sketch the graph of N against t for $0 \leqslant t \leqslant 25$.



a) At the start of the experiment t=0, so they are asking N(0):

$N(0) = 150 + 69(0) + 3(0)^2$

$N(0) = 150$ (note: it is kind of the y-intercept)

b) They are asking for N when t=5 or N(5)

$N(5) = 150 + 69(5) + 3(5)^2$

$N(5) = 570$

c) they are asking what is t, when N=2000.

$2000 = 150 + 69t + 3t^2$

Now you need to solve this equation.

1st step: rearrange it to standard form

$0 = -2000 + 150 + 69t + 3t^2$

$0 = -1850 + 69t + 3t^2$

2nd step: use your GDC to solve it==> $a_2=3$ and $a_1=69$ and $a_0=1850$

~~t = -38.9~~ not possible $\quad t = 15.9$ min

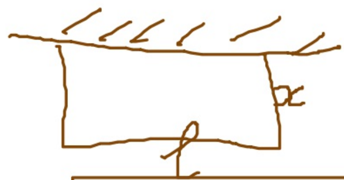
I use table from GDC

x	y
0	150
13	1554
25	3750

Ex. 6 A fence encloses a rectangular yard on three sides. The fence has a total length of 56m.

a) Write down expressions for the width and length of the yard, in terms of x.

b) Find the maximum possible area of the yard, and the corresponding dimensions.



a) $P = l + 2x$

$56 = l + 2x$

$l = 56 - 2x$

b) $A(x) = x(\underbrace{56-2x}_{l})$

$A(x) = 56x - 2x^2$

Maximum area at the vertex: $x = \frac{-b}{2a}$

$x = \frac{-56}{2(-2)}$

$x = 14$

$A(14) = 56(14) - 2(14)^2$

max $A = 392$ m^2

$l = 56 - 2(14) = 28$

the maximum area is 392 m^2 and the dimensions are 14m by 28m

Note: you could have solved this using the x-intercepts since you also have the factor form of the quadratic equation

A(x)=x(56-2x)

Hence x-intercepts are when 0=x(56-2x)

x=0 or 56-2x=0

x=-56/-2

x=28

Maximum area will happen at the vertex

$x = \frac{28+0}{2}$

$x = 14$