Calculus Optimisation Answers

1. (a) $2x + y$ (A1) 1

(b) $2500 = 2x + y$ (M1)
$2500 - 2x = y$ (AG) 1

(c) (i) Area $A(x) = xy$ (M1)
$= x(2500 - 2x)$ (M1)
$= 2500x - 2x^2$ (AG) 2

(ii) $A'(x) = 2500 - 4x$ (A1) 1

(iii) $A'(x) = 0$
$0 = 2500 - 4x$ (M1)
$4x = 2500$ (M1)
$x = 625$ (A1) 3

(iv) $A(x) = 2500x - 2x^2$
$A(625) = 2500 \times 625 - 2(625)^2$ (M2)
$= 781250$
$= 781000 \text{ m}^2$ (A1) 3
[11]

2. (a) $a = 2, b = 20, c = 9, d = 8, e = 32$ (A2) 2
Note: *Award (A2) for all 5 correct, (A1) for 3 or 4 correct, (A0) for 2 or less correct.*

(b) $A = 12x - x^2$ (C1) 1

(c) $\frac{dA}{dx} = 12 - 2x$ (A1)
A is maximum when $12 - 2x = 0$ (M1)
$\Rightarrow$ length = 6m and width = 6m (A1)
OR
length = 6m and width = 6m (A2) 3
[6]

3. (a) a = 5.30 (3s.f.) (Allow (5.30, 0) but 5.3 receives an **(AP)**.) (A1) 1

(b) $\frac{dy}{dx} = -0.042x + 1.245$ (A1)(A1) 2

Note: (A1) for each term.

(c) (i) Maximum value when $f'(x) = 0$,

$-0.042x + 1.245 = 0$, (M1)

Note: (M1) is for either of the above but at least one must be seen.

($x = 29.6$.)

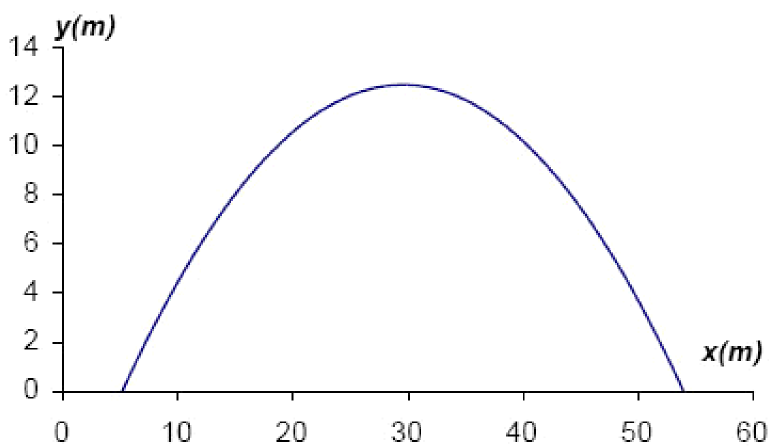
Football has travelled 29.6 – 5.30 = 24.3 m (3s.f.) horizontally. (A1)(ft)

Note: For answer of 24.3 m with no working or for correct subtraction of 5.3 from candidate's x-coordinate at the maximum (if not 29.6), award (A1)***(d)***.

(ii) Maximum vertical height, $f(29.6) = 12.4$ m (M1)(A1)(ft)(G2) 4 (UP)

Note: (M1) is for substitution into f of a value seen in part (c)(i). f(24.3) with or without evaluation is awarded (M1)(A0). *For any other value without working, award* (G0). *If lines are seen on the graph in part (d) award (M1) and then* (A1) *for candidate's value ±0.5(3s.f. not required.)*

(d) (not to scale) 4



(A1)(A1)
(A1)(ft)
(A1)(ft)

Note: Award (A1) *for labels (units not required) and scale,* (A1)*(ft) for max (29.6,12.4),* (A1)*(ft) for x-intercepts at 5.30 and 53.9, (all coordinates can be within 0.5),* (A1) *for well-drawn parabola ending at the x-intercepts.*

(e) $f(40.3) = 10.1$ m (3s.f.). (M1)(A1)(ft)(G2) 2 (UP)

Notes: Follow through from (a).

If graph used, award (M1) for lines drawn and (A1) *for candidate's value ±0.5. (3s.f. not required).*

[13]

4. (a) Area = $2(2x)x + 2xy + 2(2x)y$ (M1)(A1)

Notes: *(M1) for using the correct surface area formula (which can be implied if numbers in the correct place).*
(A1) for using correct numbers.

$300 = 4x^2 + 6xy$ (AG)2

Note: *Final line must be seen or previous (A1) mark is lost.*

(b) $6xy = 300 - 4x^2$ (M1)

$y = \frac{300-4x^2}{6x}$ *or* $\frac{150-2x^2}{3x}$ (A1)2

(c) Volume = $x\,(2x)\,y$ (M1)

$V = 2x^2\left(\frac{300-4x^2}{6x}\right)$ (A1)(ft)

$= 100\,x - \frac{4}{3}x^3$ (AG)2

Note: *Final line must be seen or previous (A1) mark is lost.*

(d) $\frac{dV}{dx} = 100 - \frac{12x^2}{3}$ or $100 - 4x^2$ (A1)(A1)2

Note: *(A1) for each term.*

(e) (i) For maximum $\frac{dv}{dx} = 0$ or $100 - 4x^2 = 0$ (M1)

$x = 5$ (A1)(ft)

$y = \frac{300-4(5)^2}{6(5)}$ or $\left(\frac{150-2(5)^2}{3(5)}\right)$ (M1)

$= \frac{20}{3}$ (A1)(ft)

(ii) $333\frac{1}{3}$ cm^3 (333 cm^3) (A1)(ft)5

(UP)

Note: *(ft) from their (e)(i) if working for volume is seen.*

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