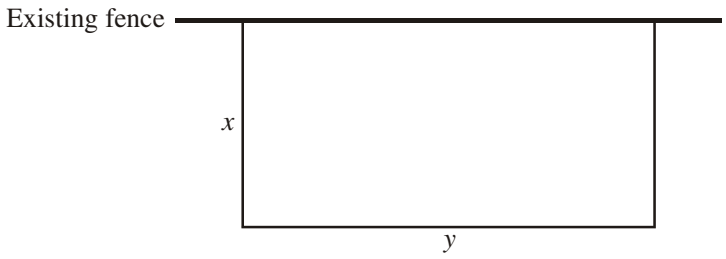
Calculus Optimisation Questions

1. A farmer wishes to enclose a rectangular field using an existing fence for one of the four sides.



(a) Write an expression in terms of x and y that shows the total length of the new fence.

(1)

(b) The farmer has enough materials for 2500 metres of new fence. Show that

$$y = 2500 - 2x$$

(1)

(c) $A(x)$ represents the area of the field in terms of x.

(i) Show that

$$A(x) = 2500x - 2x^2$$

(2)

(ii) Find $A'(x)$.

(1)

(iii) Hence or otherwise find the value of x that produces the maximum area of the field.

(3)

(iv) Find the maximum area of the field.

(3)

(Total 11 marks)

2. The perimeter of a rectangle is 24 metres.

(a) The table shows some of the possible dimensions of the rectangle.
Find the values of a, b, c, d and e.

Length (m)	Width (m)	Area (m^2)
1	11	11
a	10	b
3	c	27
4	d	e

(2)

(b) If the length of the rectangle is x m, and the area is A m^2, express A in terms of x only.

(1)

(c) What are the length and width of the rectangle if the area is to be a maximum?

(3)

(Total 6 marks)

3. A football is kicked from a point A $(a, 0)$, $0 < a < 10$ on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$$y = -0.021x^2 + 1.245x - 6.01,\ x \in \mathbb{R},\ y \geq 0.$$

x is the horizontal distance of the ball from the origin
y is the height above the ground
Both x and y are measured in metres.

(a) Using your graphic display calculator or otherwise, find the value of a.

(1)

(b) Find $\frac{dy}{dx}$.

(2)

(c) (i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum.

(ii) Find the maximum vertical height reached by the football.

(4)

(d) Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale.

(4)

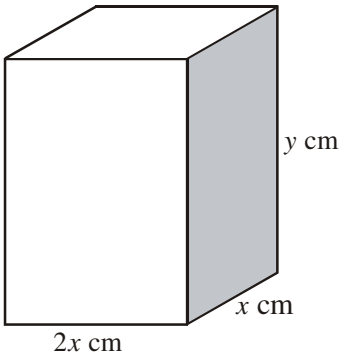
The goal posts are 35 m from **the point where the ball is kicked**.

(e) At what height does the ball pass over the goal posts?

(2)

(Total 13 marks)

4. A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2.



(a) Show that $4x^2 + 6xy = 300$.

(2)

(b) Find an expression for y in terms of x.

(2)

(c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$.

(2)

(d) Find $\frac{dV}{dx}$.

(2)

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)

(Total 13 marks)