Homework:
Page 173-4U: all
Page 174-IB style: all

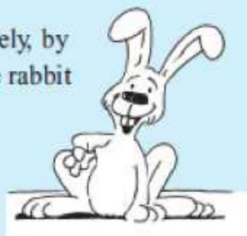
Exponential Functions: Word problems

Growth and Decay
Size becoming bigger or smaller
Temperature gain and lost.
Interest Rates
Devaluation rates

Growth

Example 3

The population size of rabbits on a farm is given, approximately, by $R = 50 \times (1.07)^n$ where n is the number of weeks after the rabbit farm was established.



- **a** What was the original rabbit population?
- **b** How many rabbits were present after 15 weeks?
- **c** How many rabbits were present after 30 weeks?
- **d** Sketch the graph of R against n $(n \geqslant 0)$.
- **e** How long it would it take for the population to reach 500?

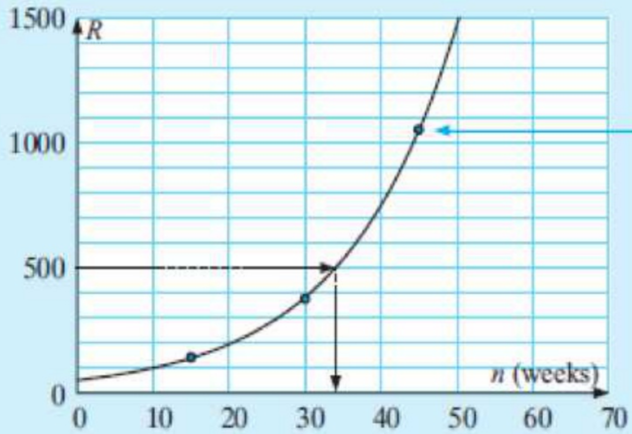
$R = 50 \times (1.07)^n$ where R is the population size and n is the number of weeks from the start.

a When $n = 0$, $R = 50 \times (1.07)^0 = 50 \times 1 = 50$ i.e., 50 rabbits originally.

b When $n = 15$, $R = 50 \times (1.07)^{15} \doteqdot 137.95$ i.e., 138 rabbits.

c When $n = 30$, $R = 50 \times (1.07)^{30} \doteqdot 380.61$ i.e., 381 rabbits.

d



when $n = 45$

$R = 50 \times (1.07)^{45}$

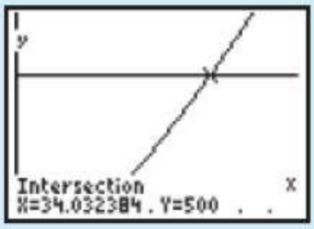
$\doteqdot 1131$

e From the graph, the approximate number of weeks to reach 500 rabbits is 34.

This solution can also be found using the solver facility of a calculator: *Answer:* $n \doteqdot 34.0$

or

by finding the intersection of $y = 50 \times (1.07)^x$ and $y = 500$ (as shown)



Decay

Example 4

When a CD player is switched off, the current dies away according to the formula $I(t) = 24 \times (0.25)^t$ amps, where t is the time in seconds.

a Find $I(t)$ when $t = 0$, 1, 2 and 3.

b What current flowed in the CD player at the instant when it was switched off?

c Plot the graph of $I(t)$ against t $(t \geqslant 0)$ using the information above.

d Use your graph and/or technology to find how long it takes for the current to reach 4 amps.

a $I(t) = 24 \times (0.25)^t$ amps

$I(0) = 24 \times (0.25)^0 = 24$ amps

$I(1) = 24 \times (0.25)^1 = 6$ amps

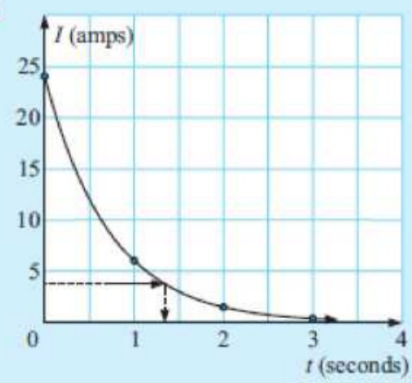
$I(2) = 24 \times (0.25)^2 = 1.5$ amps

$I(3) = 24 \times (0.25)^3 = 0.375$ amps

b When $t = 0$, $I(0) = 24$

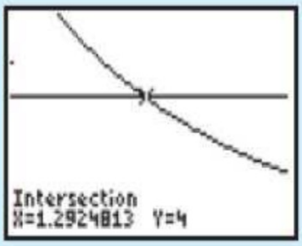
$\therefore$ 24 amps of current flowed.

c



d From the graph above, the approximate time to reach 4 amps is 1.3 seconds.

Using a calculator, the solution to 3 sig. figs. is $\doteqdot 1.29$ seconds.



Another typical example is Temperature

The temperature (°C) of a pot of water, removed from the stove after boiling, is given by $T = 18 + 82 \cdot 0.29^t$

Where "t" is the number of minutes after the pot is removed form the stove.

a) Draw the graph of temperature against time for $0 \le t \le 50$

b) Find the temperature of the water at the moment it is removed from the stove.

c) Calculate the temperature of the water 10 minutes after being removed from the stove.

d) How many minutes will have elapsed before the temperature of the water is at 40°C

e) Find the minimum possible temperature of the water.

Interest Rate (these examples could be considered *Exponential* applications or *Geometric Series* applications)

Annual

Alia deposited $2000 in the bank at 1.5% interest rate. How much would she have in 3 years.

Compounded

Dennis deposited $2000 in the bank at 1.5% interest rate compounded quarterly. How much will he have in 3 years.

2 The weight of a radioactive substance t years after being set aside is given by $W(t) = 250 \times (0.998)^t$ grams.

- **a** How much radioactive substance was put aside?
- **b** Determine the weight of the substance after:
  - **i** 400 years **ii** 800 years **iii** 1200 years.
- **c** Sketch the graph of W against t for $t \geqslant 0$, using the above information.
- **d** Use your graph or **graphics calculator** to find how long it takes for the substance to decay to 125 grams.

3 The marsupial *Eraticus* is endangered. There is only one colony remaining and survey details of its numbers have been determined at 5 year intervals:

Year	1975	1980	1985	1990	1995	2000
Number	255	204	163	131	104	84

Let n be the time since 1975 and P be the population size.

- **a** Graph P against n (with P on the vertical axis).
- **b** It is suspected that the law connecting P and n is of the form $P = a \times b^n$ where a and b are constants. Find:
  - **i** the value of a, using the 1975 data
  - **ii** the value of b, using the 2000 population data.
- **c** Check if the data for 1980, 1985, 1990 and 1995 fit the law.
- **d** In what year do you expect the population size to reduce to 50?
- **e** Use a **graphics calculator** to draw a **scatterplot** of the data graphed in **a**.
- **f** Hence find the exponential law connecting P and n. How does this law compare to the law you found in **b**?
- **g** Use **f** to check your prediction in **d**.