

# FINANCIAL MATHEMATICS 2

## COMPOUND INTEREST, DEPRECIATION & INFLATION

Chapter 7 pages 314 - 320

## COMPOUND INTEREST

• Compound interest is where the interest is added to the principal each period so that interest is paid on it in future (interest on interest).

Compound interest formula:

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

*booklet*

Where FV = Future Value (amount in bank after interest)  
 PV = Present Value (Principal, what you start with)  
 r = annual interest rate (nominal) %  
 n = numbers of years  
 k = number of compounding periods (how often the bank pays interest)

## NOMINAL INTEREST RATE

- Compound interest is sometimes paid more than once a year.
- Typical periods are half yearly ( $\frac{2}{1 \times yr}$ ), quarterly ( $\frac{4}{1 \times yr}$ ), monthly ( $\frac{12}{1 \times yr}$ ) or even daily ( $\frac{365}{1 \times yr}$ ).
- The advertised interest rate will still be the yearly one, you need to calculate the rate for each time period interest is paid.
- The yearly interest rate paid this way is called the nominal interest rate.

## Worked Examples

1. Find the total interest earned for the following compound interest investments.
- 950 euro after 2 years at 5.8% p.a. with interest calculated annually.  $k=1$
  - 17,500 after 4 years at 8.4% p.a. with interest calculated annually.

Interest earned = FV - PV

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

a)  $FV = 950 \left(1 + \frac{5.8}{100(1)}\right)^{(1)2}$  interest =  $1063.40 - 950 = \boxed{\text{€}113.40}$   
 $= 1063.40 \text{ €}$

b)  $FV = 17500 \left(1 + \frac{8.4}{100(1)}\right)^{(1)4}$  interest =  $24163.24 - 17500 = \boxed{\text{€}6663.24}$   
 $= \text{€}24163.24$

## USING THE CALCULATOR: TI NSPIRE

Calculator document: menu → 8: Finance → 1: Finance Solver

You will see this dialog box, where

- N: the total number of payments  
 I(%): the annual interest rate as a percent  
 PV: the present value, which is negative for investments  
 Pmt: the payment or regular deposit, which is negative for investments = 0  
 FV: the future value  
 PpY: the payments per year = 1  
 CpY: the interest calculations period per year = k  
 PmtAt: payments made at the end or beginning of each period = END

**Note: PV should be entered as a negative value**

Enter all the variables, then put cursor on the unknown, in this case, FV. Press ENTER.

N	4
I%	8.4
PV	-17500 <i>must be (negative)</i>
Pmt	0 = 0
FV	?
PpY	1
CpY	1
PmtAt	END

Press ENTER to calculate Future Value: FV

## Worked Examples

4. Giovanni's bank manager told him that if he invests 3000 EUR now, compounding yearly, it will be worth 4600 EUR in 5 years' time. What is the rate of interest p.a.?

Show working:

- substitute into formula
- copy the TVM table from GDC

$n = 5$   
 $I\% = ?$   
 $PV = -3000$   
 $FV = 4600$   
 $PpY = 1$   
 $CpY = 1$

$$FV = PV \left(1 + \frac{r}{100k}\right)^{kn}$$

$$4600 = 3000 \left(1 + \frac{r}{100}\right)^5$$

$r = \boxed{8.92\%}$

### Example to Try

5. Marina is saving to buy a small boat that costs 35,000 USD. She has 28,000 in an account that pays 5.34% interest compounding yearly. How long must Marina wait before she can buy the boat?

5 years

### Worked Examples

6. a) The bank in Grabiton is advertising a nominal yearly rate of 5% with compounding applied quarterly. State the number of compounding periods for a 3-year investment and find the actual interest rate applied each time period.  
b) Fleur invests 500 GBP in this bank for three years. Calculate the total amount in her account after this time.  
c) Suppose Fleur invests this rate of 5% p.a. compounding only once a year. How much less interest would she receive?

a) 1.25%

b)  $FV = 578.81$

c) £ 1.57 less

### Example to Try

7. A bank in Australia is offering a "term deposit" account with a choice of interest rates. As long as you leave your money with them for 2 years, you can get a nominal rate of:  
a) 5.05% compounded monthly or  
b) 5.10% compounded quarterly or  
c) 5.15% compounded half yearly or  
d) an actual yearly rate of 5.2%.  
Abigail has 7000 AUD to invest. Which is her best option? c)

### Example to Try

8. Annoushka has 2736.74 EUR in her bank account. She has left her money there for exactly 3 years at a nominal rate of 4.1% p.a. compounding daily. Calculate, correct to the nearest EUR, how much Annoushka put in the account when she opened it. (Assume there were no leap years in that time).

$PV = 2420.02$

### Example to Try

9. For how long must Martin invest \$5000 at 6.45% p.a. compounded half-yearly if he wants to double his money?

11 years

$n = 10.92$  years

$N = ?$   
 $I = 6.45$   
 $PV = -5000$   
 $PMT = 0$   
 $FV = 10,000$   
 $P/Y = 1$   
 $C/Y = 2$

## DEPRECIATION

- When something loses value, we say it depreciates.
- Common examples include the value of a car or other expensive item that will eventually need to be replaced. Businesses include depreciation as part of their long term budgets.
- Depreciation is calculated using the compound interest formula, but  $r$  is now negative.

### Worked Examples

1. Vijay has paid 300,000 Indian rupees (INR) for a car. The car depreciates at a rate of 9% p.a.
- Calculate the value, V, of the car in 4 years time, giving your answer correct to the nearest INR.
  - Find the percentage loss over the 4-year period.

$N = 4$   
 $I = -9$   
 $PV = 300000$   
 $PMT = 0$   
 $FV = ?$   
 $P/Y = 1$   
 $C/Y = 1$

$FV = 205,724.88$   
 $FV = 205,725 \text{ INR}$

$300000 - 205725 = 94275 \text{ INR}$   
 $\% \text{ total loss} = \frac{94275}{300000} \times 100 = 31.4\%$

### Example to Try

2. Mary has some shares in a telecommunications company. She paid 7.50 AUD per share 3 years ago but now each share is only worth 3.90 AUD. Calculate the depreciation rate over the 3 years.

$N = 3$   
 $I = ? \Rightarrow -19.59$   
 $PV = 7.50$   
 $PMT = 0$   
 $FV = 3.90$   
 $P/Y = 1$   
 $C/Y = 1$

$\text{depreciation rate} = 19.59\%$   
 $19.6\% \text{ 3 sf.}$

### Example to Try

3. Anthony bought a house for 380,000 USD 7 years ago. Since then, in his area, houses have increased in value by an average of 10% p.a. For the first 5 years, but then lost value at a rate of 4% p.a. for the last 2 years. What is the value, V, of the house now? Give your answer to the nearest 1000 USD. Do this in 2 steps

1st 5 years  
 $N = 5$   
 $I = 10$   
 $PV = -380000$   
 $PMT = 0$   
 $FV = ? \rightarrow \$611993.80$   
 $P/Y = 1$   
 $C/Y = 1$

last 2 years  
 $N = 2$   
 $I = -4$   
 $PV = 611993.80$   
 $PMT = 0$   
 $FV = ? \rightarrow \$564013.49$   
 $P/Y = 1$   
 $C/Y = 1$   
 $= \$564000$

## INFLATION

- The price of goods tend to go up every year - this is inflation.
- It is measured by the Consumer Price Index (CPI) which annually calculates the price of buying a selection of goods & services (the same each year).
- Inflation takes away some of the value of investments (because you can't buy as much with the same amount of money).
- The real return & the real rate of return are the ways of finding out how much you have gained after the effect of inflation.

### Example to Try

6. From June 2005 to June 2006, the Australian Bureau of Statistics recorded a CPI of 4.4%.
- If a half-litre of milk cost 2.05 AUD in June 2005, how much did it cost in June 2006?
  - Based on the same CPI, how much will the milk cost in June 2007?
  - How much will it cost in June 2010?

$N = 1$   
 $I = 4.4$   
 $PV = -2.05$   
 $PMT = 0$   
 $FV = ? \rightarrow 2.14 \text{ 2006}$   
 $P/Y = 1$   
 $C/Y = 1$

$N = 2$   
 $FV = ? \rightarrow 2.23 \text{ 2007}$

$N = 5$   
 $FV = ? \rightarrow 2.54 \text{ 2010}$

### Example to Try

7. Based on the same 4.4% annual inflation rate, calculate what the cost (to the nearest dollar) of a new car would have been in June 2005, if the price in June 2007 is 22,000 AUD.

$N = 2$   
 $I = 4.4$   
 $PV = ? \rightarrow \$20184.67$   
 $PMT = 0$   
 $FV = 22000$   
 $P/Y = 1$   
 $C/Y = 1$

$= \$20185$

# **HOMEWORK**

Exercises 74, p320: odds

71, p320: odds