Quadratics

1

Factoring Expressions

Remember, what do we do with expressions?

☐ simplify them

☑ factorize them

Expressions?

What do we do with expressions?

- ✓ Simplify them
- ✓ Factorize them

What kind of expressions are you familiar with?

- ✓ Linear $2x+5$
- ✓ Exponential 2^{x+1}
- ✓ Quadratic x^2-2x+1
- ✓ Reciprocal $\frac{1}{2x}$
- ✓ Trigonometric expression??? $\sin(2x)$
- ✓ Radical $\sqrt{x+1}$

What are the tools we have in Math for Simplifying.....

- ☑ Order of operations
- ☑ Like terms
- ☑ Exponent Rules
- ☑ Distributive prop.

What is the purpose of factoring?

- You can use factoring
  to simplify expressions
  See next slide
- solve equation:
  refer to next presentation

Showing you the big picture!!!

This is **not** in your tests and/or quizzes. However, it is to show you the one of the purpose of factoring. You did this in grade 9Ext.

Simplify $\Rightarrow$ $\dfrac{2x+2}{x+1}$

$\dfrac{2(x+1)}{(x+1)}$

$\boxed{2}$

Simplify $\Rightarrow$ $\dfrac{x^2-x+2}{x-2}$

$\dfrac{(x+1)(x-2)}{(x-2)}$

$\boxed{x+1}$

Simplify $\Rightarrow$ $\sqrt{x^2-2x+1}$

$\sqrt{(x-1)^2}$

$\boxed{x-1}$

Polynomial expressions: What are they?

$$ax^n + bx^{n-1} + cx^{n-2} \ldots + d$$

Most textbooks (including IB) use this form

or

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \ldots + a_0$$

Your GDC (NSpire and TI-84) use this one.

Which two polynomial expressions are you familiar with?

Linear

$2x-5$

Here $n=1$

$a=2$ & $b=-5$

or

$a_1=2$ & $a_0=-5$

Quadratic

$2x^2-5x+3$

Here $n=2$

$a=2,\ b=-5,$ & $c=3$

or

$a_2=2,\ a_1=-5,$ & $a_0=3$

Factoring Flow Chart

Quadratic Expression

Or linear for that matter!!

Remove Common Factors

In some cases, you are done here!!!

Recognize Type & factor accordingly

1st Remove Common Factors

$21x^2 - 7x$

$7x(3x - 1)$

$3x^2 - 9x + 12$

$3(x^2 - 3x + 4)$

$4x^2 - 6x + 2$

$2(2x^2 - 3x + 1)$

Difference of Two Squares: $a^2 - b^2$

You should be able to recognize these cases:

1. It is a binomial and not a trinomial
2. It is ALWAYS a difference (not a sum)
3. Both terms are "perfect squares"

Identity:

$a^2 - b^2 = (a - b)(a + b)$

Example: $25x^2 - 9$

- ☑ it is a binomial
- ☑ a difference (- sign)
- ☐ terms are perfect squares

$(5x)^2$ and $(3)^2$

$\therefore 25x^2 - 9$

$(5x - 3)(5x + 3)$

Difference of Two Squares: $a^2 - b^2$

More examples:

$4x^2 - 49$

$(2x - 7)(2x + 7)$

$16x^4 - 81$

$(4x^2 - 9)(4x^2 + 9)$

$9x^2 - y^2$

$(3x - y)(3x + y)$

These two expressions are not Quadratic expressions!

Just tricking you!

Trinomials with a = 1: $ax^2 + bx + c$

example ① x^2+4x+3 example ② x^2-4x+3

1. With trinomials where a = 1, you always start with $(x\quad)(x\quad)$

ex ① $(x\quad)(x\quad)$ ex ② $(x\quad)(x\quad)$

2. when the last term has a +ve sign, then you have two options
$(x-\ )(x-\ )$ or $(x+\ )(x+\ )$. You need to look at the middle term to decide which option.

ex ① $(x+\ )(x+\ )$ ex ② $(x-\ )(x-\ )$

3. Now you need to look at the factors that make term "c" and add up to "b"

our "c" in both ex ① & ② is 3

Factors of 3 ⇒ 3x1 and they add to "b"

which is 4 (3+1)

ex 1 $(x+1)(x+3)$ ex 2 $(x-1)(x-3)$

More examples:

① $x^2+7x+10$ → 10 → 10x1 but 10+1≠7; 5x2 good 5+2=7

$(x+5)(x+2)$

② $x^2-8x+12$ → 12 → 12x1 but 12+1≠8; 3x4 but 3+4≠8; 6x2 Yes! 6+2=8

$(x-6)(x-2)$

③ $x^2+8x+16$ → 16 → 16x1 but 16+1≠8; 8x2 but 8+2≠8; 4x4 Yes! 4+4=8

$(x+4)(x+4)$ or $(x+4)^2$

Note: if you remember your perfect squares Identities, then you should have seen it was a perfect square. $(a+b)^2 = a^2+2ab+b^2$

Rules for Signs in Factors

Another type: example ① $x^2-4x-12$ Example ② $x^2+4x-12$

1. With trinomials where a=1, you should start with $(x\quad)(x\quad)$

ex ① $(x\quad)(x\quad)$ Ex ② $(x\quad)(x\quad)$

2. When the last term of the trinomial "c" has a -ve sign, then you know $(x-\quad)(x+\quad)$

Ex ① $(x-\quad)(x+\quad)$ Ex ② $(x-\quad)(x+\quad)$

3. Now you look for factors of "c", but they must subtract to get "b"

c is 12 → 12x1 but 12-1≠4; 3x4 but 3-4≠4; 6x2 Yes! 6-2=4

4. Now, the trickiest part. Which factor goes with the -ve sign and which factor goes with the +ve sign? the greater factor goes with the same sign as "b"

Ex ① $(x-6)(x+2)$ sign of $-4x$ Ex ② $(x-2)(x+6)$ sign of $+4x$

Trinomials with a = 1: $ax^2 + bx + c$

Examples

① $x^2+2x-15$ → 15 → 15x1 but 15-1≠2; 3x5 Yes!

$(x-3)(x+5)$ ⇒ greatest factor!

② $x^2-14x-15$ → 15 ⇒ 15x1 Yes!

$(x-15)(x+1)$

③ x^2-x-12 → 12 → 12x1 but 12-1≠1; 6x2 but 6-2≠1; 3x4 Yes! 4-3=1

$(x-4)(x+3)$

Trinomials with a ≠ 1: $ax^2 + bx + c$

There are several methods to teach factoring when a≠1, but I find splitting the x-term the easiest. example: $2x^2+11x+12$

1. Multiply "a" times "c", and find the factors that when you add, you get the x-term

$a=2$ and $c=12$ ∴ $a \times c = 24$ → 24×1 but $24+1 \neq 11$; 12×2 but $12+2 \neq 11$; 8×3 → yes! $8+3=11$

2. Rewrite the trinomial, but split the x-term using the factors from part 1.

$2x^2+8x+3x+12$ Note: 8x + 3x is the same as 11x

3. Group the first two terms and the last two.

$(2x^2+8x)+(3x+12)$

4. Factor the first group and then the second group. You will see a common factor

$2x(x+4)+3(x+4)$

5. Factor out the common term, and voila!!!

$(x+4)(2x+3)$

Trinomials with a ≠ 1: $ax^2 + bx + c$

It's get tricky with a - sign. Be careful!!!!

Example: $3x^2+x-2$

① $a=3$, $c=-2$ } $a \times c = -6$

$6 \times (-1) \Rightarrow 6-1 \neq 1$

$-6 \times 1 \Rightarrow 1-6 \neq 1$

$3 \times (-2) \Rightarrow 3-2=1$

$(-3) \times 2 \Rightarrow 2-3 \neq 1$

② split the x-term with the factors from part 1. $3x^2+3x-2x-2$

③ group but be careful → Here is where it gets tricky $(3x^2+3x)-(2x+2)$

④ Factor $3x(x+1)-2(x+1)$

⑤ $(3x-2)(x+1)$