


Quadratics

2

Solving Equations

What does mean to solve an equation?
Find the value(s) of x that satisfy the equation!

HOW
~~What~~ do you know it is a quadratic equation? 

Linear

$$2x - 5 = 15$$

$$2x = 20$$

$$x = 10$$

Usually 1 solution

Sometimes "No solution"

example

$$2(x+1) = 2x - 5$$

$$2x + 2 = 2x - 5$$

$$2 = -5$$

Does not make sense \therefore No solutions

Sometimes "ALL solutions"

example $2(x+1) = 2x + 2$
 $2x + 2 = 2x + 2$
All solutions or $0 = 0$

Quadratic

$$ax^2 + bx + c = 0$$

$$2x^2 - 5x + 3 = 0$$

Here $a = 2, b = -5$ & $c = 3$

Could $a = 0$?

No! it would
Not be a quadratic.
it would be linear!

Could $b = 0$?

Yes! No
x-term

Could $c = 0$?

Yes! No indep (or constant)
term

Solutions:

Usually 2 solutions

Sometimes ONE

Sometimes "NO solution"

Solving Methods

Solving methods:

Isolating x^2 term

Factoring

Completing the square (Only needed for HL)

Quadratic Formula

GDC

When $b=0$, isolate the x^2

$$x^2 + c = 0$$

example

①

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

②

$$x^2 = 9$$

$$x = \pm 3$$

③

$$x^2 + 36 = 0$$

$$x^2 = -6$$

Stop No solution!

Solving by FACTORING



Note: you may have to rearrange the equation to the standard form

Quadratic Expression

Note: if the expression is factorable, then there is always at least one solution. You cannot have NO Solution.

Remove Common Factors

Recognize Type & factor accordingly

When $c=0$, FACTOR!!!!



$$ax^2 + bx = 0$$

these are the easiest to factor!! At least, you always have the "x" as a common factor!

example: $21x^2 - 7x = 0$

$$7x(3x - 1) = 0$$

$$\therefore 7x = 0$$
$$\boxed{x = 0}$$

$$\text{or } 3x - 1 = 0$$
$$\boxed{x = \frac{1}{3}}$$

Note: when $c=0$ always one of the solutions is $x=0$

Difference of Two Squares: $a^2 - b^2$

Recognise this type of factoring! Refer to previous presentation:

- It is a binomial ✓
- It is a difference (- sign) ✓
- Both terms are perfect squares ✓ $(5x)^2$ and $(6)^2$

example $25x^2 - 36 = 0$

$$(5x - 6)(5x + 6) = 0$$

$$5x - 6 = 0 \quad \text{or} \quad 5x + 6 = 0$$

$$5x = 6$$

$$\boxed{x = \frac{6}{5}}$$

$$5x = -6$$

$$\boxed{x = -\frac{6}{5}}$$

Trinomials with $a = 1$: $ax^2 + bx + c$

You obviously need to know how to factor to solve by factoring
Once you factor, the rest is easy. Either one factor equals zero or the other one. So, solve the two linear (easy) equations and done!!

examples

1st step
2nd step
Last step

factor

$$x^2 + 3x + 2 = 0$$

$2 = 2 \times 1$
and
 $2 + 1 = 3$

$$(x + 2)(x + 1) = 0$$

Now solve each linear factor.

$$x + 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$\boxed{x = -2} \quad \boxed{x = -1}$$

Trinomials with $a = 1$: $ax^2 + bx + c$

example

$$x^2 - 6x + 9 = 0$$

$$(x - 3)(x - 3) = 0$$

or maybe you saw $(x - 3)^2 = 0$
either way $\Rightarrow x - 3 = 0$

$$x = 3$$

ONE solution

example

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 3$$

$$x = -2$$

Trinomials with $a \neq 1$: $ax^2 + bx + c$

$$4x^2 - 10x - 6 = 0$$

$$4x(-6) = -24 \quad \rightarrow \quad 8x(-3) \Rightarrow 8-3 \neq -10$$

$$\rightarrow (12)x(2) \Rightarrow -12+2 = -10 \checkmark$$

$$(4x^2 - 12x) + (2x - 6) = 0$$

$$4x(x - 3) + 2(x - 3) = 0$$

$$(x - 3)(4x + 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad 4x + 2 = 0$$

$$x = 3$$

$$x = \frac{-2}{4} = -\frac{1}{2}$$

Trinomials with $a \neq 1$: $ax^2 + bx + c$

$$3x^2 - 17x + 10 = 0$$

$$a \times c \Rightarrow 3 \times 10 = 30$$

$$\begin{aligned} &\rightarrow 3 \times 10 \text{ but } 3 + 10 \neq 17 \\ &\rightarrow 5 \times 6 \text{ but } 5 + 6 \neq 17 \\ &\text{or } (-5) \times (-6) \text{ but } -5 - 6 \neq -17 \\ &\rightarrow (-15) \times (-2) \text{ yes! } -15 - 2 = -17 \end{aligned}$$

Be careful
grouping

$$3x^2 - 15x - 2x + 10 = 0$$

$$(3x^2 - 15x) - (2x - 10) = 0$$

$$3x(x - 5) - 2(x - 5) = 0$$

$$(x - 5)(3x - 2) = 0$$

$$x - 5 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$\boxed{x = 5}$$

$$\boxed{x = \frac{2}{3}}$$

Trinomials with $a \neq 1$: $ax^2 + bx + c$

$$25x^2 + 10x + 1 = 0$$

Can you see the perfect square?

$$(5x + 1)^2 = 0$$

$$\therefore 5x + 1 = 0$$

$$\boxed{x = -\frac{1}{5}}$$

If you do not see it, don't worry. Just factor normal way.

$$25x^2 + 5x + 5x + 1 = 0$$

$$5x(x + 1) + 1(5x + 1) = 0$$

$$(5x + 1)(5x + 1) = 0$$

The QUADRATIC FORMULA



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

$$x^2 - x - 4 = 0$$

$$\begin{aligned} a &= 1 \\ b &= -1 \\ c &= -4 \end{aligned}$$

1. First, you need to decide what is a, b, and c
Remember, you may need to rearrange the equation into Standard Form
2. Then, you plug into the formula
3. Simplify the formula to its simplest radical

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1+16}}{2}$$

$$x = \frac{1 + \sqrt{17}}{2}$$

$$x = \frac{1 - \sqrt{17}}{2}$$

The QUADRATIC FORMULA



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

$$4x^2 + 2x + 1 = 0$$

$$\begin{aligned} a &= 4 \\ b &= 2 \\ c &= 1 \end{aligned}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{4-16}}{8}$$

$$x = \frac{-2 \pm \sqrt{-12}}{8}$$

STOP
No solution!

$$3x - 5 = \frac{4}{x}$$

Rearrange first

$$\begin{aligned} x(3x - 5) &= 4 \\ 3x^2 - 5x &= 4 \end{aligned}$$

$$3x^2 - 5x - 4 = 0$$

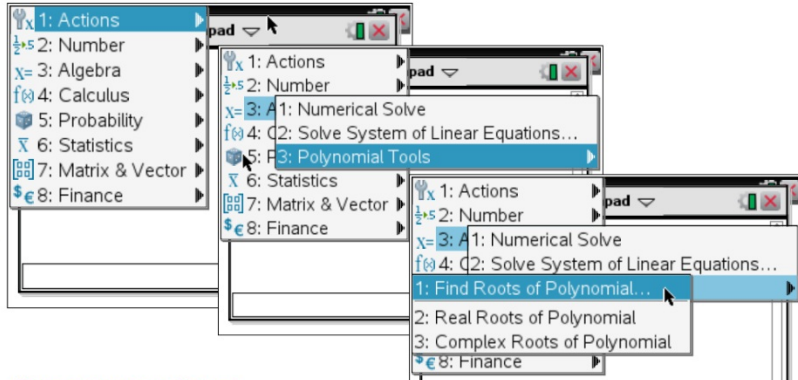
Now you can start

$$\begin{aligned} a &= 3 \\ b &= -5 \\ c &= -4 \end{aligned}$$

Let's solve $x^2 - x - 2 = 0$, we already know from factoring $(x+1)(x-2)=0$ that the solutions are $x=-1$ & $x=2$
 But let's use our GDC!

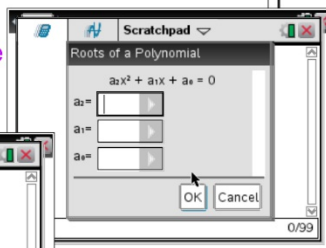
FIRST: make sure you are in scratchpad.

- 2: Hit menu
- 3: hit #3 (Algebra)
- 4: hit #3(Polynomial Tools)
- 5: hit #1(Find Roots of Polyn....)



Once you are here:
 you want degree 2 for
 quadratics, and you want
 real solutions. HIT OK

Now, you are
 ready to enter the
 coefficients



Be careful with
 negative numbers
 Do not enter subtraction

