3D-Geometry:
Surface Area and Volume

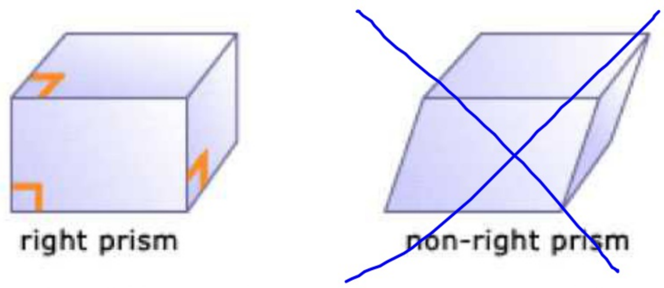
Surface Area:

Lateral Area + Area of bases

Volume:

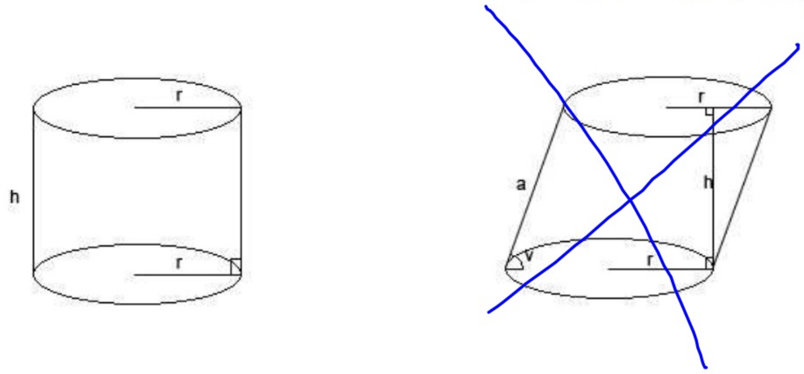
Area of base × Height

Right prisms/cylinder versus non-right



Rectangular Prism = 2 rectangular bases + 4 lateral faces

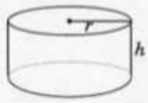
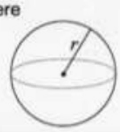
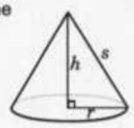
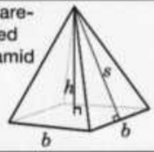
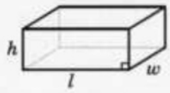
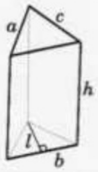
Not in Math Studies



My advise is: do not follow formulas! Just decode!!!!

- **Surface Area:** the sum of the areas of all the faces/visible surfaces of a 3D shape
- **Volume:** the amount of space an object occupies.
- **Net:** 2D representation of of the faces of a 3D figure

Check your Formulae Booklet to know which formulas are given

Geometric Figure	Surface Area	Volume
Cylinder 	$A_{base} = \pi r^2$ $A_{lateral\ surface} = 2\pi rh$ $A_{total} = A_{2\ bases} + A_{lateral\ surface}$ $= 2\pi r^2 + 2\pi rh$	$V = (A_{base})(\text{height})$ $V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$
Cone 	$A_{lateral\ surface} = \pi rs$ $A_{base} = \pi r^2$ $A_{total} = A_{lateral\ surface} + A_{base}$ $= \pi rs + \pi r^2$	$V = \frac{(A_{base})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$A_{triangle} = \frac{1}{2}bs$ $A_{base} = b^2$ $A_{total} = A_{4\ triangles} + A_{base}$ $= 2bs + b^2$	$V = \frac{(A_{base})(\text{height})}{3}$ $V = \frac{1}{3}b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$A = 2(wh + lw + lh)$	$V = (\text{area of base})(\text{height})$ $V = lwh$
Triangular prism 	$A_{base} = \frac{1}{2}bl$ $A_{rectangles} = ah + bh + ch$ $A_{total} = A_{rectangles} + A_{2\ bases}$ $= ah + bh + ch + bl$	$V = (A_{base})(\text{height})$ $V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$

Surface Area and Volume: "general rule"

Surface area = Lateral Area + (2 × Area of the BASE)

S or S.A.= Lateral Area + 2B

Where S or S.A. = cm² or m² or Km²

B = area of the BASE (2D) = cm² or m², Km²

Volume = Area of the BASE X Height

V = B X h

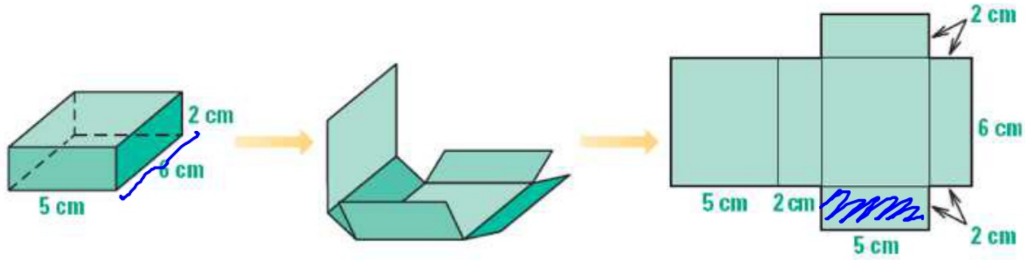
Where V = cm³, m³, or mm³

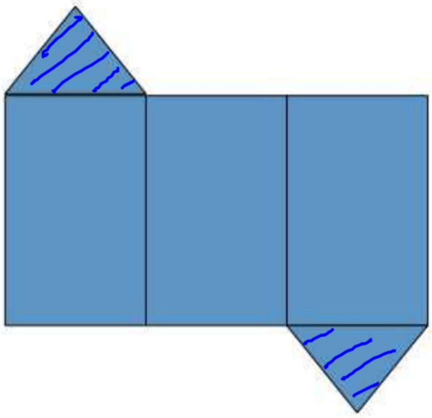
B = area of Base (cm², m² ...)

h = height of the Solid

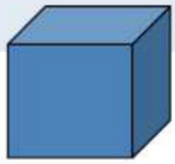
note: B is the area of the Base, not a length.

Nets and Surface Areas





Right Prisms



1. Cube: S=6 FACES (all squares) 6 x (area of one face)

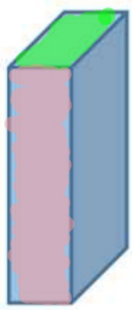
   Volume: (Area of face) x Side

2. Square based Prisms

   S=2 Bases( ) + 4 lateral faces ( ) = $[2 \times [s^2]] + [4 \times [s \times h]]$

   s^2 $l \times w$

   Volume: (Area of Base square □) x h



3. Rectangular Prims→ S=2 Bases( ) + 2 lat. Faces (

   $(l \times w)$ lw

   +2 lateral faces (

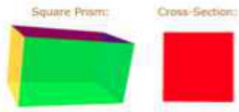
   $l \times w$

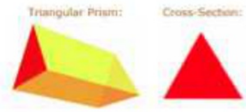
   Volume:

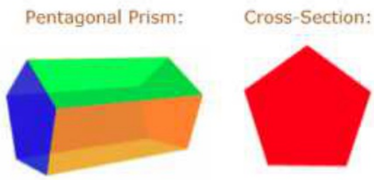
   Area of Base $(l \times w) \times h$

Right Prisms

- Cube
- Square base prism
- Rectangular base prism
- Triangular base prism
- Pentagonal base prism
- Hexagonal base prism
- Octagonal base prism
- Etc……





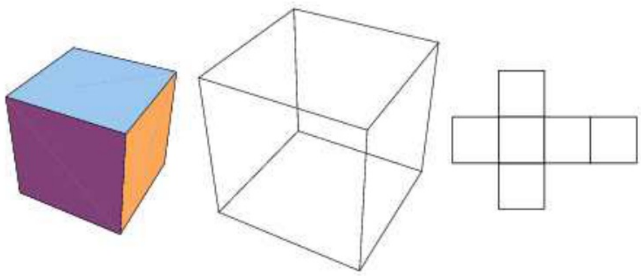


CUBE

- Surface Area:

$S.A. = 6 \times \square$

$= 6 \times [5^2]$



- Volume:

$V = \text{Base} \times H$

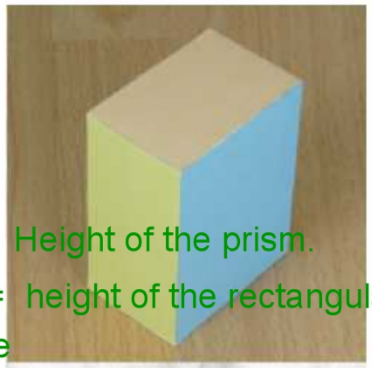
$= (5^2) \times 5$

$= 5^3$

CUBOID (square or rectangular prism)

- Square base prism:
  - Surface Area: S. A= 2 Bases + 4 Lateral

    They are all congruent
  - Volume: $V = S^2 \times H$
- Rectangular base prism:



  - Surface Area:

    $S = 2\text{ Bases} + 2\text{ Lateral}_1 + 2\text{ Lateral}_2$
  - Volume: $V = b \cdot h \cdot \mathcal{H}$ or $w \cdot l \cdot \mathcal{H}$

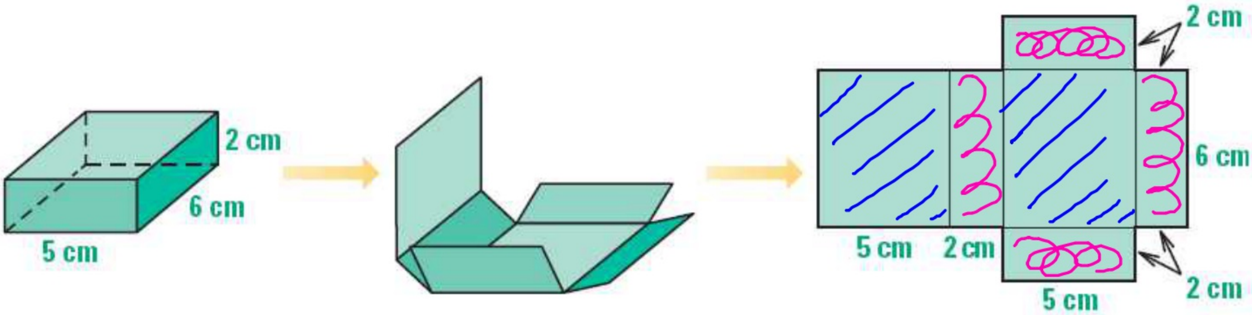
    BASE

$\mathcal{H}$ = Height of the prism.

h = height of the rectangular base

Example: Examine (decode) every surface.

- Find the area of the rectangles that form the faces of the prism. It is almost as if you unfold the solid to make a net.



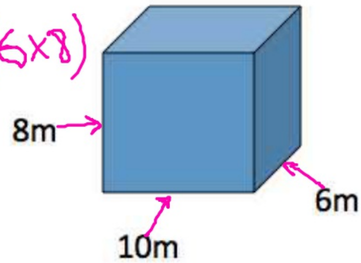
lateral

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12\ cm^2$
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10\ cm^2$
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30\ cm^2$

Example: Volume and Surface Area

$S.A = [2 \times \text{Base}] + [2 \times \text{lateral}] + [2 \times \text{Lateral}]$

$= 2 \times (10 \times 6) + 2 \times (8 \times 10) + 2(6 \times 8)$



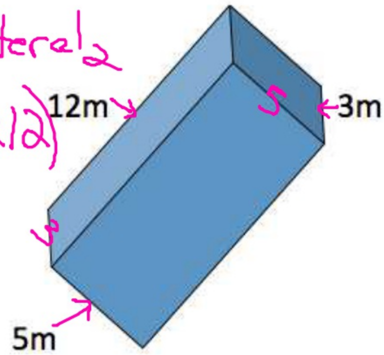
$\text{Base} \times h$

$\text{Volume} = (10 \times 6) \times 8$

$= 480 \text{ m}^3$

$S.A. = 2 \times \text{Base} + 2 \times \text{lateral}_1 + 2 \text{ lateral}_2$

$= 2 \times (5 \times 3) + 2(12 \times 5) + 2(3 \times 12)$



$\text{Volume} = \text{Base} \times h$

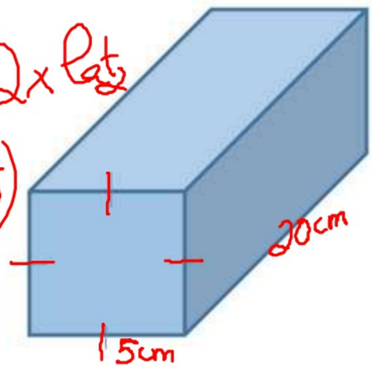
$= (5 \times 3) \times 12$

$=$

Trick question: Square based or rectangular based prism?

Answer: It is up to you!!!!!

$S.A = 2 \times \text{Base} + 2 \times \text{lateral}_1 + 2 \times \text{lat}_2$



One way: rectangular prism.

$S.A. = 2(20 \times 5) + 2(5 \times 5) + 2(20 \times 5)$

$= 200 + 50 + 200$

$= 450 \text{ cm}^2$

$V = \text{Base} \times h \Rightarrow V = (20 \times 5) \times 5 = 500 \text{ cm}^3$

Another way: Square base prism

$S.A = 2(5 \times 5) + 4(20 \times 5)$

$= 50 + 400 = 450 \text{ cm}^2$

$V = \text{Base} \times h \Rightarrow V = (5 \times 5) \times 20$

$= 500 \text{ cm}^3$

Triangular base prism

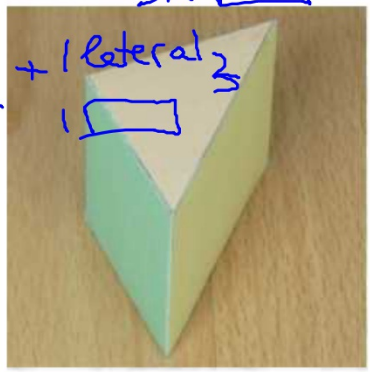
- 3 types of base: $2 \times \text{Base} + 3\ \text{Lateral}$

  $2 \times \triangle \qquad 3 \times \square$

  - Scalene triangle

    $S.A = 2\ \text{Bases} + 1\ \text{Lateral}_1 + 1\ \text{lateral}_2 + 1\ \text{lateral}_3$

    $2\triangle \qquad 1\square \qquad 1\square \qquad 1\square$



  - Isosceles triangle

    $S.A = 2\ \text{Bases} + 1\ \text{lateral}_1 + 2\ \text{lateral}_2$

    $2\triangle \qquad 1\square \qquad 2\square$

  - Equilateral triangle

    $S.A. = 2 \times \text{Bases} + 3\ \text{lateral}$

    $2\triangle \qquad 3\square$

Example: Triangular prisms

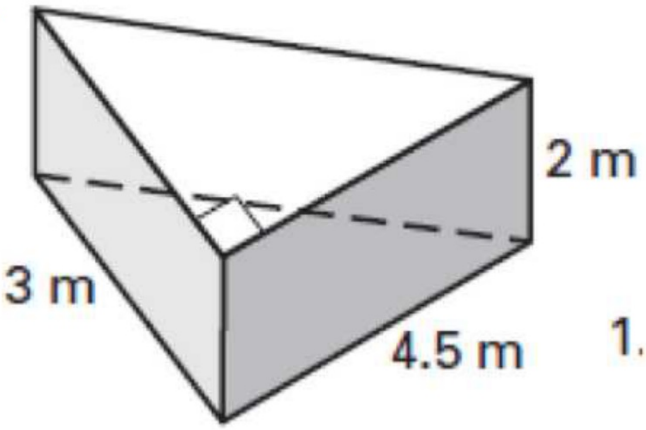
Base

$A = \frac{b \times h}{2}$

$A = \frac{3 \times 4.5}{2}$

Height

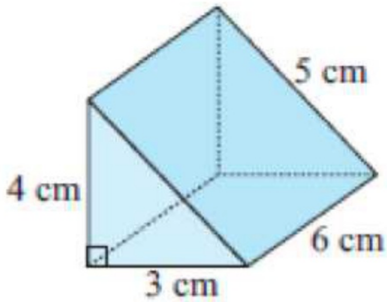
2



1.

Triangular prism can be tricky

2 Find the total surface area of:



Base

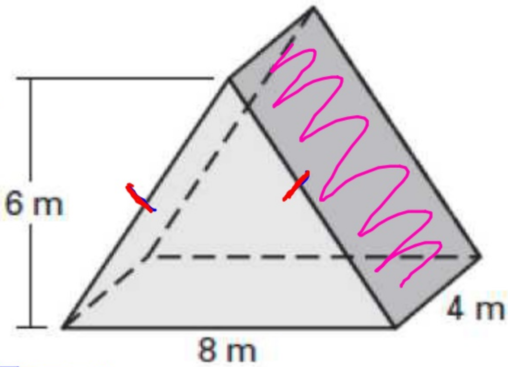
$\left(\frac{4 \times 3}{2}\right)$

Height

6

Therefore Volume = 6 cm^2 • 6 cm
= 36 cm^3

Δ is isosceles in this case. **<u>do not assume</u>** is always isosceles.



Base

Height

4

6

8m

4

$?^2 = 6^2 + 4^2$

$?^2 = 36 + 16$

$?^2 = 52$

$? = 7.21$

$$S.A = 2\left(\frac{6 \times 8}{2}\right) + \left(8 \times 4\right) + 2\left(4 \times 7.21\right)$$

$$= 48 + 32 + 57.68 = 137.68\ m^2$$

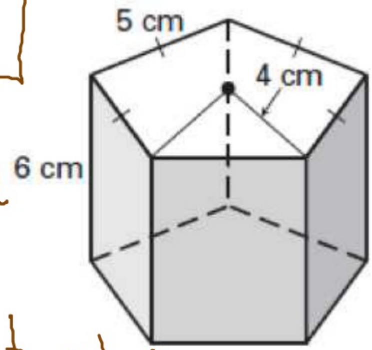
$$\text{Volume} = \left(\frac{6 \times 8}{2}\right) \times 4 = 96\ m^3$$

Other Polygon bases

Lateral Area $= 5\,\square$

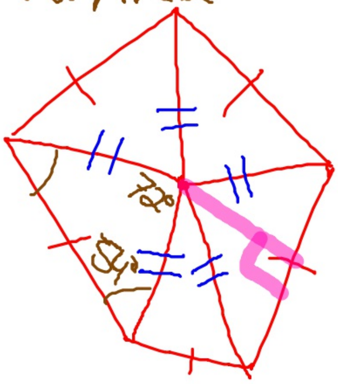
$= 5[6 \times 5]$

$= 150\,cm^2$



A.Base $= 5 \times \triangle$

S.A = 2 Base + lateral Area



$A = \frac{b \times h}{2}$

$A = \frac{5 \times 3.12}{2}$

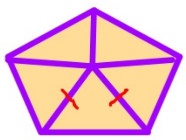
$A = \frac{1}{2}(4)(5)\sin 54$

Other regular polygons, besides equilateral ∆s and squares.

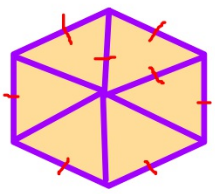
Note1: When you get pentagons, hexagons, heptagons, octagons...etc... you should divide the polygon into congruent triangles and then multiply by the number of triangles to calculate the total area.

eg. Area of Octagonal base = Area of 1∆ X 8

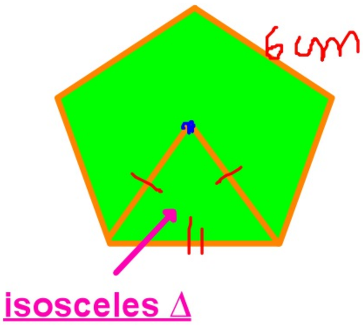
Note 2: Don't forget that the triangles made inside the regular polygons are all isosceles!!!! Why do you think???

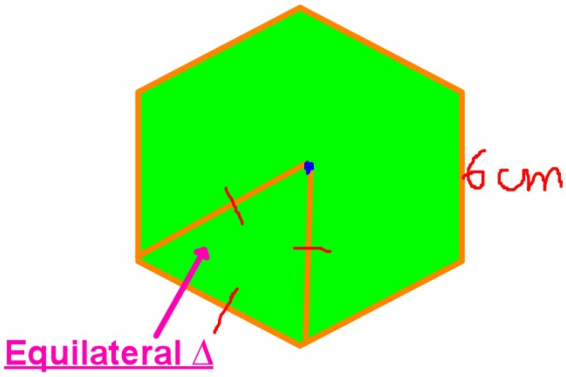


Note 3: When working with Regular HEXAGONS (6 sides), the triangles formed are EQUILATERAL. only with hexagons!!!!

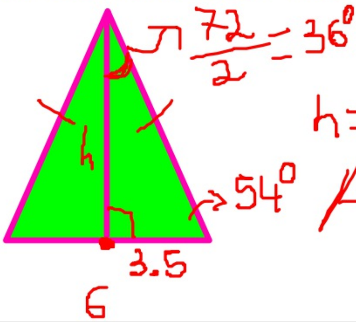


Remember that when a **<u>regular</u>** (all sides and angles are congruent) polygon is divided into congruent Δs, the Δs are **<u>isoscele</u>**s except for the Hexagon (6 sides) where the 6 congruent Δs are **<u>equilateral</u>**





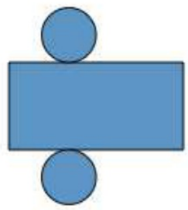
Now, to find the Area of the polygon, get the area of 1Δ and multiply by the number of them (i.e. Pentagon--> by 5). The question is how do you get the area of that Δ. There are several methods. For example:



$h = 3.5 \tan 54°$

$A =$

Cylinders



- Surface area: 2 bases ( ) + 1 lateral face ($l \times w$)

  $[2 \times (\pi r^2)] + (2\pi r) \times h$

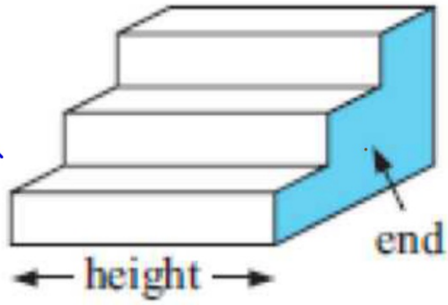
- Volume: Base X Height

  $(\pi r^2) \times h$

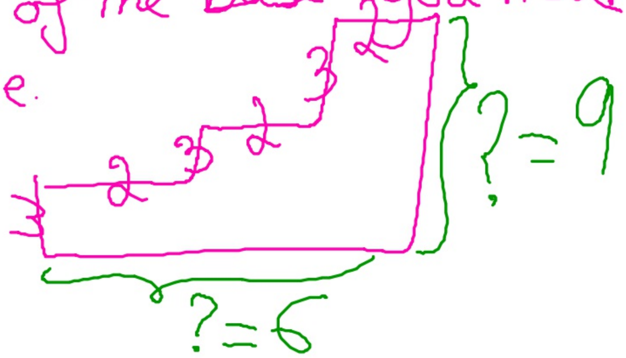
Composite solids: decode!!!

Can you figure it out?

It would be a prism if you consider the blue area the Base

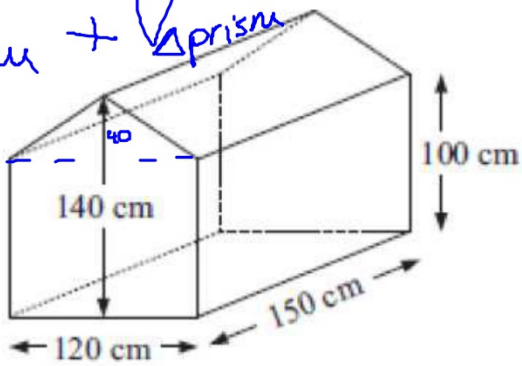


to get the area of the base you need many lengths. i.e.



Composite solids

- Typical: $\text{Volume} = V_{\text{prism } \square} + V_{\Delta \text{prism}}$
  - A house Base



$V_{\square \text{ prism}} = [100 \times 120] \times 150$
$= 1800000 \text{ cm}^3$

$V_{\Delta \text{prism}} = \left[\frac{120 \times 40}{2}\right] \times 150$
$= 360000 \text{ cm}^3$

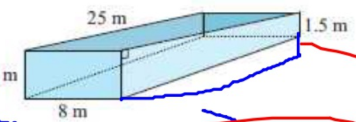
$V =$

  - A pool

$V_{\text{pool}} = V_{\square} - V_{\Delta}$

Rect. prism

Δ prism



$= [(8 \times 3) \times 25] - \left[\left(\frac{25 \times 1.5}{2}\right) \times 8\right]$

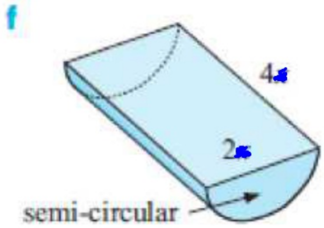
$3 - 1.5 = 1.5$

Composite Solids

- Semi-circular:

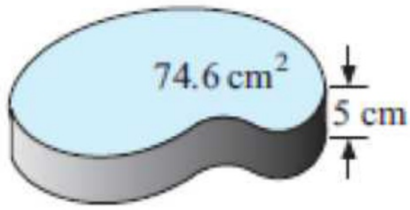
$$\text{Volume} = \frac{1}{2} V$$
$$= \frac{1}{2}\left[\pi r^2 \times h\right]$$
$$= \frac{1}{2}\left[\pi (1)^2 \times 4\right]$$

$$S.A = 2D + \frac{1}{2} \boxed{} + \boxed{}$$
$$= 2\left[\frac{1}{2}\pi r^2\right] + \frac{1}{2}\left[2\pi r \times 4\right] + 4 \times 2$$

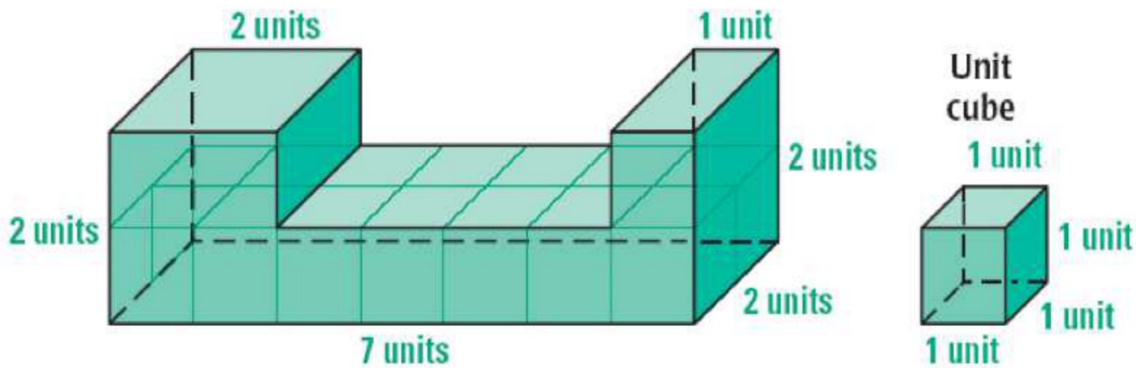


- Irregular shape:

Volume only $= 74.6 \times 5$



Composite solids

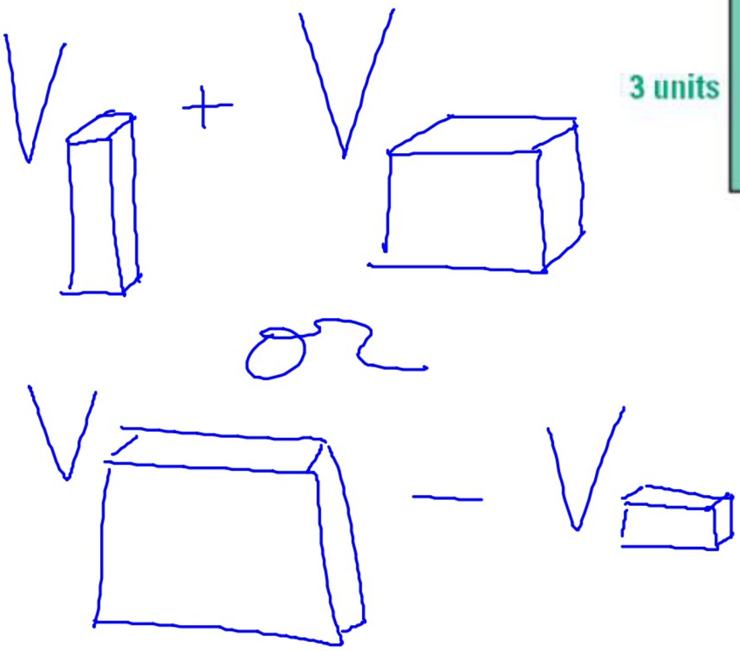


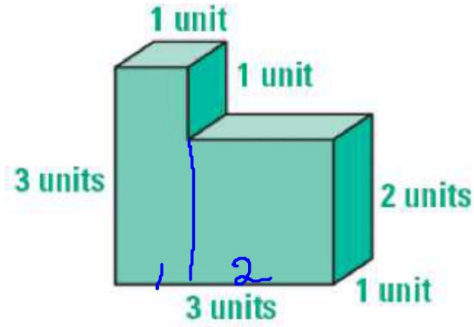
$$V_{total} = V + V + V$$

or

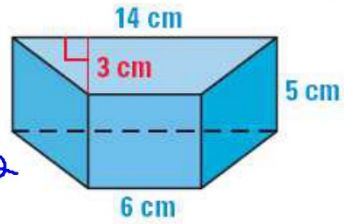
$$V_{total} = V - V$$

Volume: 2 ways



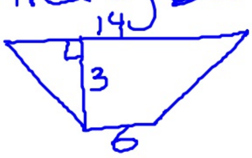


What kind of prism is this? trapezoidal prism



① Volume

Area of Base $= \left(\frac{b_1 + b_2}{2}\right) \times h$



$= \left(\frac{14+6}{2}\right) \times 3 = 30\ cm^2$

$\therefore$ Volume = Base × H

$= 30 \times 5 = 150\ cm^3$