Geometric Sequences & Series

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Geometric Sequences

- Definition

A sequence is Geometric if each term can be obtained from the previous one by multiplying by the same non-zero constant (number)

i.e. 2, 10, 50, 250, … $\Longrightarrow r=5$

$\times 5 \quad \times 5 \quad \times 5$

- The number/constant that is multiplied is called the constant ratio (r)

Therefore r= $\frac{u_n}{u_{n-1}}$ or $\frac{u_3}{u_2}$ or $\frac{u_{10}}{u_9}$

The formulas…

- Given a, b, and c are any consecutive terms of a geometric sequence then

$$\frac{b}{a} = \frac{c}{b} \rightarrow b = \sqrt{ac}$$

$$b^2 = ac$$

- The general term: $\Rightarrow \boxed{u_n = u_1 r^{(n-1)}}$

how

$$u_2 = u_1 \times r$$

$$u_3 = u_2 \times r = u_1 \times r \times r = u_1 r^2$$

$$u_4 = u_3 \times r = u_1 r^2 \times r = u_1 r^3$$

Examples

- Find the 8th term of this geometric sequence: 24, 12, 6, 3, …

$r = \frac{1}{2}$ $\quad u_1 = 24$

$$u_8 = 24\left(\frac{1}{2}\right)^7 = 0.1875$$

- Find k given that k-1, 2k, and 21-k are consecutive terms of a geometric sequence.

We did a similar example with arithmetic seq. and we also had 2 methods to solve.

Method 1:
using the concept/definition of a geometric sequence.

$$\frac{2k}{(k-1)} = \frac{(21-k)}{2k}$$

$$4k^2 = (21-k)(k-1)$$

$$4k^2 = (21-k)(k-1)$$

$$4k^2 = 21k - 21 - k^2 + k$$

$$5k^2 - 22k + 21 = 0$$

$$k = 3$$

$$k = \frac{7}{5}$$

Geometric Mean

$$(2k)^2 = \left(\sqrt{(21-k)(k-1)}\right)^2$$

$$4k^2 = 21k - 21 - k^2 + k$$

Examples $u_n = u_1 r^{(n-1)}$

Remember: two terms ====> 2 equations.

- The second term of a geometric is -15 and the fifth term is 405. Find the first term and common ratio.

$u_2 = -15 \Longrightarrow$ $u_2 = u_1 r$ equation 1

$-15 = u_1 r$

$u_5 = 405$

$u_5 = u_1 r^4$ equation 2

$405 = u_1 r^4$

$$u_1 = \frac{-15}{r}$$

$$405 = \left(\frac{-15}{\cancel{r}}\right) r^{\cancel{4}\,3}$$

$$405 = -15 r^3$$

$$\frac{405}{-15} = r^3$$

$$\sqrt[3]{-27} = \sqrt[3]{r^3}$$

$$\boxed{r = -3}$$

from before $u_1 = \frac{-15}{r}$

$$u_1 = \frac{-15}{-3}$$

$$\boxed{u_1 = 5}$$

Examples

$u_n = u_1 r^{(n-1)}$

a) Find the general term of the geometric sequence

6, $6\sqrt{2}$, 12, $12\sqrt{2}$

$$\frac{6\sqrt{2}}{6} = \sqrt{2} \longrightarrow r = \sqrt{2}$$

$$u_1 = 6$$

$$\frac{12}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{6 \times 2} = \frac{12\sqrt{2}}{12} = \sqrt{2}$$

$$\boxed{u_n = 6(\sqrt{2})^{n-1}}$$

b) Find the first term that exceeds 1400

$$u_n = 1400$$

$$1400 = 6(\sqrt{2})^{(n-1)}$$

$$n = 16.7324\ldots$$

Examples

- A geometry sequence has $u_2 = -6$ and $u_5 = 162$. Find its general term This means you must find the common ratio and the 1st term

system $\begin{cases} -6 = u_1 r \quad (1) \\ 162 = u_1 r^4 \quad (2) \end{cases}$ solve by substitution from (1) $u_1 = \frac{-6}{r}$

Now subs (1) into (2)

$162 = \left(\frac{-6}{r}\right) r^4$

$162 = -6r^3$

$\sqrt[3]{-27} = \sqrt[3]{r^3}$

$\boxed{-3 = r}$

then solve for u_1 with $r = -3$

$u_1 = \frac{-6}{-3} \Rightarrow \boxed{u_1 = 2}$

the general term $u_n = 2(-3)^{(n-1)}$

- Find k given that k, $k+8$ and $9k$ are successive terms fo a geometric sequence.

Using definition

$\frac{k+8}{k} = \frac{9k}{k+8}$

$(k+8)^2 = (k)(9k)$

$k^2 + 16k + 64 = 9k^2$

$-8k^2 + 16k + 64 = 0$

you could solve by factoring (easy!)
However, I would always use GDC!!!

using geometric mean

$(k+8)^2 = \left(\sqrt{(k)(9k)}\right)^2$

$k^2 + 16k + 64 = 9k^2$

$-8k^2 + 16k + 64 = 0$

use GDC to solve

$k = -2$ or $k = 4$

This is an important note: in previous example, we solved for r first. r = -3 and then for $u_1 = 2$

When you write the general term for THIS sequence, make sure to put ( ) around a negative ratio.

$$u_n = 2(-3)^{(n-1)}$$

if you write

$$\cancel{u_n = 2 - 3^{(n-1)}}$$

it is ambiguous.