Geometric Sequences & Series IB Questions

**1.** The seventh term, *u*7, of a geometric sequence is 108. The eighth term, *u*8, of the sequence is 36.

(a) Write down the common ratio of the sequence.

(1)

(b) Find *u*1.

(2)

 The sum of the first *k* terms in the sequence is 118 096.

(c) Find the value of *k.*

(3)

(Total 6 marks)

**2.** A geometric sequence has 1024 as its first term and 128 as its fourth term.

(a) Show that the common ratio is .

(2)

(b) Find the value of the eleventh term.

(2)

(c) Find the sum of the first eight terms.

(3)

(d) Find the number of terms in the sequence for which the **sum** first exceeds 2047.968.

(3)

(Total 10 marks)

**3.** A geometric sequence has second term 12 and fifth term 324.

(a) Calculate the value of the common ratio.

(4)

(b) Calculate the 10th term of this sequence.

(3)

(c) The *k*th term is the first term that is greater than 2000. Find the value of *k*.

(3)

(Total 10 marks)

**4.** The annual fees paid to a school for the school years 2000, 2001 and 2002 increase as a geometric progression. The table below shows the fee structure.

|  |  |
| --- | --- |
| **Year** | **Fees (USD)** |
| 2000 | 8000.00 |
| 2001 | 8320.00 |
| 2002 | 8652.80 |

(a) Calculate the common ratio for the increasing sequence of fees.

(2)

 **In parts (b) and (c) give your answer correct to 2 decimal places.**

 The fees continue to increase in the same ratio.

(b) Find the fees paid for 2006.

(2)

 A student attends the school for eight years, starting in 2000.

(c) Find the **total** fees paid for these eight years.

(2)

(Total 6 marks)

**5.** A geometric progression *G*1 has 1 as its first term and 3 as its common ratio.

(a) The sum of the first *n* terms of *G*1 is 29 524. Find *n*.

(3)

A second geometric progression *G*2 has the form 1, , ,  …

(b) State the common ratio for *G*2.

(1)

(c) Calculate the sum of the first 10 terms of *G*2.

(2)

(d) Explain why the sum of the first 1000 terms of *G*2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures.

(1)

(e) Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence 2, 3 , 9 , 27  …

Give your answer **correct to one decimal place**.

(3)

(Total 10 marks)

Geometric Sequences & Series IB Questions- Answers

**1.** (a) *r* =  (A1) (C1)

**Note:** Accept 0.333.

(b) *u*1 = 36 (M1)

**Note:** Award (M1)for correct substitution in formula for nth term of a GP. Accept equivalent forms.

 *u*1 = 78732 (A1)(ft) (C2)

**Notes:** Accept 78700. Follow through from their common ratio found in part (a). If 0.333 used from part (a) award (M1)(A1)(ft)for an answer of 79285 or 79300 irrespective of whether working is shown.

(c) 118096 =  (M1)(M1)

**Notes:** Award (M1)for correct substitution in the sum of a GP formula, (M1)for equating their sum to 118096. Follow through from parts (a) and (b).

 **OR**

 Sketch of the function *y* = 78732 (M1)
Indication of point where *y* = 118 096 (M1)

 **OR**

 78 732 + 26 244 + 8748 + 2916 + 972 + 324 + 108 + 36 + 12 + 4
= 118 096 (M1)(M1)

**Note:** Award (M1)for a list of at least 8 correct terms, (M1)for the sum of the terms equated to 118096.

 *k* = 10 (A1)(ft) (C3)

**Notes:** Follow through from parts (a) and (b). If k is not an integer, do not award final (A1). Accept alternative methods.
If 0.333 and 79285 used award (M1)(M1)(A1)(ft)for k = 5.
If 0.333 and 79300 used award (M1)(M1)(A0).

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**2.** (a) 1024*r*3 = 128 (M1)

 *r*3 =  **or** *r* =  (M1)

 *r* =  (0.5) (AG)

**Notes:** Award at most (M1)(M0)if last line not seen.
Award (M1)(M0)if 128 is found by repeated multiplication (division) of 1024 by 0.5 (2).

(b) 1024 × 0.510 (M1)

**Notes:** Award (M1)for correct substitution into correct formula.
Accept an equivalent method.

 1 (A1)(G2)

(c) *S*8 =  (M1)(A1)

**Note:** Award (M1)for substitution into the correct formula, (A1)for correct substitution.

 **OR**

 (A1)for complete and correct list of eight terms (A1)
(M1)for their eight terms added (M1)

 *S*8 = 2040 (A1)(G2)

 (d)  *>* 2047.968 (M1)(M1)(ft)

**Notes:** Award (M1)for correct substitution into the correct formula for the sum, (M1)for comparing to 2047.968.
Accept equation. Follow through from their expression for the sum used in part (c).

 **OR**

 If a list is used: *S*15 = 2047.9375 (M1)
 *S*16 = 2047.96875 (M1)

 *n* = 16 (A1)(ft)(G2)

**Note:** Follow through from their expression for the sum used in part (c).

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**3.** (a) *u*1*r*4 = 324 (A1)
*u*1*r* = 12 (A1)
*r*3 = 27 (M1)
*r* = 3 (A1)(G3)

**Note:** Award at most (G3) for trial and error.

(b) 4 × 39 = 78732 **or** 12 × 38 = 78732 (A1)(M1)(A1)(ft)(G3)

**Note:** Award (A1) for u1 = 4 if n = 9, **or** u1 = 12 if n = 8, (M1) for correctly substituted formula. (ft) from their (a).

 (c) 4 × 3*k*–1 > 2000 (M1)

**Note:** Award (M1) for correct substitution in correct formula. Accept an equation.

 *k* > 6 (A1)
*k* = 7 (A1)(ft)(G2)

**Notes:** If second line not seen award (A2) for correct answer. (ft) from their (a).
Accept a list, must see at least **3** **terms** including the 6th and 7th.

**Note:** If arithmetic sequence formula is used consistently in parts (a), (b) and (c), award (A0)(A0)(M0)(A0) for (a) and (ft) for parts (b) and (c).

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**4.** ***Financial penalty (FP) applies in parts (b) and (c)***

(a) *r* =  (or equivalent) (M1)

**Note:** Award (M1) for dividing correct terms.

 *r* = 1.04 (A1) (C2)

**Notes:** In (b) and (c) (ft) from candidate’s r.
Allow lists, graphs etc. as working in (b) and (c).

(b) Fees = 8000 (1.04)6 (M1)

**Note:** Award (M1) for correct substitution into correct formula.

**FP** Fees = 10122.55 USD (USD not required) (A1)(ft) (C2)

**Note:** Special exception to the note above.
Award maximum of (M1)(A0) if 5 is used as the power.

(c) Total =  (M1)

**Notes:** Award (M1) for correct substitution into correct formula.
Give full credit for solution by lists.

**FP** Total = 73713.81 USD (USD not required) (A1)(ft) (C2)

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**5.** (a)  (M1)(A1)3

**Notes:** (M1) for correctly substituted formula on one side, (A1) for = 29524 on the other side.

*n* = 10. (A1)(G2)

**Note:** Trial and error is a valid method. Award (M1) for at least  seen and then (A1) for = 29524, (A1) for n = 10.

For only unproductive trials with n  10, award (M1) and then (A1) if the evaluation is correct.

 (b) Common ratio is  (A1) 1

**Note:** Accept ‘divide by 3’.

 (c)  (M1)

= 1.50 (3s.f.) (A1)(ft)(G1)2

**Notes:** 1.5 and  receive (A0)**(AP)** if **AP** not yet used

Incorrect formula seen in (a) or incorrect value in (b) can follow through to (c).

Can award (M1) for 

(d) Both  and  (or those numbers divided by 2/3) are 0 when
corrected to 3s.f., so they make no difference to the final answer. (R1)1

**Notes:** Accept any valid explanation but please **Note:** statements which only convey the idea of convergence are not enough for (R1). The reason must show recognition that the convergence is adequately fast (though this might be expressed in a much less technical manner).

 (e) The sequence given is G1 + G2 (M1)

The sum is 29 524 + 1.50 (A1)(ft)

= 29 525.5 (A1)(ft)(G2) 3

**Notes:** The (M1) is implied if the sum of the two numbers is seen. Award (G1) for 29 500 with no working.

(M1) can be awarded for  Award final (A1) only for answer given correct to 1dp.

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