

Geometric Series

Homework: page 307-7E: 1 to 6

Remember: $r \neq 0$ and $r \neq 1$. It wouldn't make sense.

$$S_n = u_1 + u_2 + u_3 + u_4 + \dots + u_n$$

- The FORMULA:

$$S_n = \frac{u_1(r^n - 1)}{r - 1} \text{ when } r > 1$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r} \text{ when } r < 1$$

Examples

- Find the sum of $2 + 6 + 18 + 54 + \dots$ to 12 terms.

S_{12} ? $u_1=2$
 $r=3$

$$\begin{aligned} S_{12} &= \frac{u_1(r^n - 1)}{r - 1} \\ &= \frac{2(3^{12} - 1)}{3 - 1} \end{aligned}$$

$$= 531440$$

- Find the general formula for S_n for $9 - 3 + 1 - \frac{1}{3} + \dots$ to n terms

$$u_1 = 9 \quad r = -\frac{1}{3} \quad 9 + (-3) + 1 + (-\frac{1}{3}) + \dots$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r} = \frac{9(1 - (-\frac{1}{3})^n)}{1 + \frac{1}{3}}$$

$$S_n = \frac{9(1 - (-\frac{1}{3})^n)}{1 + \frac{1}{3}}$$

$$S_n = \frac{9 - 9(-\frac{1}{3})^n}{\frac{4}{3}}$$

$$S_n = \frac{3(9 - 9(-\frac{1}{3})^n)}{4}$$

$$S_n = \frac{27 - 27(-\frac{1}{3})^n}{4}$$

Examples

- A geometry progression has the form 6, 2, 2/3...

- State the common ratio for this sequence
- Calculate the sum of the first 10 terms of this seq.

a) $r = \frac{1}{3}$

b) $S_{10}?$ $S_n = \frac{u_1(1-r^n)}{1-r}$

$$S_{10} = \frac{6(1 - (\frac{1}{3})^{10})}{1 - \frac{1}{3}} = 8.99998$$

$$S_{10} = 9.00$$

Examples

- A ball is dropped vertically. It reaches a height of 1.6 m of the first bounce. The height of each subsequent bounce is 80% of the previous bounce.

- a) Find the height the ball reaches on the 6th bounce.
b) Find the sum of the first seven terms of this sequ.

$$1.6, 1.28, 1.024, \dots \quad r = 0.8 \text{ or } \frac{4}{5}$$

a) $u_n = u_1 r^{(n-1)}$
 $u_6 = 1.6 \cdot (0.8)^5$
 $u_6 = 0.524 \text{ m}$

b) Next slide

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

$$S_7 = \frac{1.6(1-(\frac{4}{5})^7)}{1-(\frac{4}{5})}$$

$$S_7 = 6.322278\dots$$

$$\boxed{S_7 = 6.32}$$

Examples

- Penelope is starting her first job. She will earn

\$24 000 in the 1st year and her salary will increase by 4% every year. Calculate how much Penelope will earn in her 4th year of work.

$$r = 1.04 \quad u_1 = 24000$$

$u_4?$

Do you understand where I get $r = 1.04$?

$$u_4 = u_1 r^3$$

$$u_4 = 24000 (1.04)^3$$

$$u_4 = 26996.736$$

$$= 27000$$