Logic

1: Symbolic Logic & Truth Tables

p380 – 394

Symbolic Logic

- "logic" comes from the Greek for "word".
- Logic in maths deals with the conversion of word statements to symbols, and the use of those symbols to make deductions and create proofs.
- mathematical logic deals with statements called propositions.
- propositions may be true or false.
- They are NOT a comment / question / exclamation.

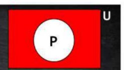
Propositions

- The answer (T or F) may depend on the situation (= indeterminate)
- Notation: Use letters eg p, q, r (usually the end of the alphabet)

Eg: Which of these are propositions?

a) All dogs have tails. Prop F
b) Freddy is handsome. ? prop – indeterminate.
c) It is raining today. prop – indeterimate
d) $3^2 = 9$ prop T
e) $4^2 = 21$ prop F
f) $y = 6$ not a prop – because y is not defined
g) For all $x \in \mathbb{R}$, $x^2 \geq 0$. prop T

Negations



- The negation of a proposition is its negative. $p' = \neg p$
- Symbol = $\neg$
- If p = proposition, $\neg p$ = it's negation or "not p"

Eg p = "It is raining" (False) $\neg p$ = "It is not raining" (True)

- Can have a "double" negation = $\neg(\neg p)$

= it is not (not raining) = it is raining

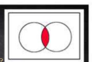
Therefore p is the same as $\neg(\neg p)$

- negation corresponds to the complement in sets

- The domain (the possible set of values a variable belongs to) can determine the value of a negation.

Eg x is a dog, for $x \in$ {dogs, cats} so $\neg x$ is a cat

Conjunction = AND



*corresponds to intersection

- Compound propositions are statements which join propositions.
- When two propositions are joined with AND, the new proposition is the conjunction of the original propositions.
- Notation: If p and q are propositions, $p \wedge q$ is their conjunction ("p and q")
- Truth Table for $p \wedge q$:

the conjunction is only true if both propositions are also true.

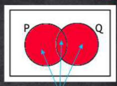
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction = OR

- When two propositions are joined with OR, the new proposition is the disjunction of the original propositions.
- Notation: If p and q are propositions, $p \vee q$ is their inclusive disjunction ("either p or q or both is true")
- If p and q are propositions, $p \underline{\vee} q$ is their exclusive disjunction ("only p or q is true")

Inclusive Disjunction = OR

- $p \vee q$ means p or q or both p and q.



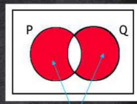
- Truth Table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Eg: p: Mischa has a cat
q: Yuri has a dog
$p \vee q$: Mischa has a cat or Yuri has a dog (& both could be true)

Exclusive Disjunction = OR

- $p \veebar q$ means p or q but not both.



- Truth Table:

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

Eg: p: Mischa has a cat
q: Yuri has a dog
$p \veebar q$: Mischa has a cat or Yuri has a dog (but NOT both are true)

Worked Examples

3. Given: w: I am wearing shorts
s: I am going to swim
r: I am going to run

Write in words these propositions:

a) $w \wedge s$ b) $s \veebar r$ c) $w \wedge \neg(s \vee r)$

a) I am wearing shorts and I am going to swim

b) I am going to swim or run, but not both

c) I am wearing shorts and I am not going to run or swim

"neither / nor"

Worked Examples

4. If x: Sergio would like to go swimming tomorrow, and
y: Sergio would like to go bowling tomorrow, *write down in symbolic form*:

a) Sergio would not like to go swimming tomorrow $\neg x$

b) Sergio would like to go swimming and bowling tomorrow
$x \wedge y$

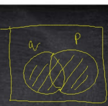
c) Sergio would like to go swimming or bowling tomorrow $x \vee y$

d) Sergio would not like to go both (swimming and bowling) tomorrow $\neg(x \wedge y)$

e) Sergio would not like to go swimming and go bowling tomorrow $\neg x \wedge y$

g) Sergio would like either to go swimming or go bowling, but not both, tomorrow $x \veebar y$

Truth Tables



- Combining all the truth tables into one:

p	q	Negation $\neg p$	Conjunction $p \wedge q$	Inclusive Disjunction $p \vee q$	Exclusive Disjunction $p \veebar q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

- Truth tables can be taken as definitions of these statements.

* These are in your booklet

Truth Tables for 3 propositions

- Truth tables for 3 propositions need to have eight possibilities of True/False

4/4 2/2 1/1

- List the T/F values in a systematic way every time.

Eg: Make the truth table for: $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

Worked Examples

3. Draw a truth table involving three propositions *p*, *q*, and *r*. Construct a column for the compound proposition $(p \vee q) \wedge (\neg p \vee r)$.

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	—	T	T	T
F	T	F	—	T	T	—
F	F	T	T	F	T	F
F	F	F	T	=	T	F

Homework

Exercise 9C p386, Q 1, 3, 8;
9D p389, Q 1, 3, 5, 11;
9E p392, Q 2, 4, 7