Logic

3: Converse, Inverse & Contrapositive

p409 – 412

Converse

- The converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$.

Eg: For p: the triangle is isosceles, and q: two angles of the triangle are equal, state $p \Rightarrow q$ and its converse $q \Rightarrow p$.

$p \Rightarrow q$ is if the triangle is isosceles, then two angles of the triangle are equal.

$q \Rightarrow p$ is if two angles of the triangle are equal, then the triangle is isosceles.

Eg: What is the converse of "if $x > 10$, then $x > 4$"?

p: $x > 10$, and q: $x > 4$. The statement is $p \Rightarrow q$.

The converse is $q \Rightarrow p$, and is "if $x > 4$, then $x > 10$" (this is F)

Inverse

$T \Rightarrow F = F$

- The inverse of the statement $p \Rightarrow q$ is the statement $\neg p \Rightarrow \neg q$.
- truth table:

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

- Note that $\neg p \Rightarrow \neg q$ is the same truth table as $q \Rightarrow p$ so these are logically equivalent.
- The converse and the inverse of an implication are logically equivalent.

Contrapositive

$T \Rightarrow F$ F

- The contrapositive of the statement $p \Rightarrow q$ is the statement $\neg q \Rightarrow \neg p$.
- truth table:

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

- Note that $\neg q \Rightarrow \neg p$ is the same truth table as $p \Rightarrow q$ so these are logically equivalent.
- The implication and its contrapositive are logically equivalent.

Worked Examples

1. Consider s: If my shoes are too small then my feet hurt.
   Write in words, the converse, the inverse and the contrapositive of s.

converse: if my feet hurt then my shoes are too small

Inverse: If my shoes aren't too small then my feet won't hurt!

contrapositive: if my feet do not hurt, then my shoes are not too small.

Analysing Arguments

- Break down the argument into propositions.
- Connect the propositions to make a compound statement. (Use AND to join sentences without other obvious clues). "therefore" $\Rightarrow$
- Test the validity of the argument with a truth table. (tautology = valid).
- Note – you are analysing the STRUCTURE of the argument, not whether the "words" make sense or not!

Worked Examples

2. Analyse the following argument:

   If Dubai wins the expo, I will visit Dubai. I am not visiting Dubai, therefore Dubai won't win the expo.

p = Dubai wins the expo

q = I will visit Dubai

$$\left[(p \Rightarrow q) \wedge (\neg q)\right] \Longrightarrow \neg p$$

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \neg q$	$\left[(p \Rightarrow q) \wedge \neg q\right] \Rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

tautology

$\therefore$ the argument is valid

Example to Try

2. Analyse the following argument:

   If I study for my mocks, I will pass my exams. If I pass my exams, I will go to my top university. Therefore, if I study for my mocks, I will go to my top university.

p: I study for my mocks

q: I pass my exams

r: I will go to a top university

$$\left[(p \Rightarrow q) \wedge (q \Rightarrow r)\right] \Rightarrow (p \Rightarrow r)$$

Homework

Exercise 9J p406, Q 2, 4, 6;

9M p411, odds – instruction # 2 only