

Mathematical Studies Syllabus

Topic 1: Number and Algebra

- Know the number sets & their definitions: Natural numbers \mathbb{N} , Integers \mathbb{Z} , Rational numbers \mathbb{Q} , and Real numbers \mathbb{R}
- Know how to give numbers to an approximate value using either a specified number of decimal places or significant figures.
- Calculate percentage error
- Estimate values to 1 sf and know if the result is reasonable
- Express numbers in the form of $a \times 10^k$, where $1 \leq a < 10$ and k is an integer ($k \in \mathbb{Z}$).
- Perform operations with numbers in scientific notation (as above).
- Be familiar with SI and other basic units of measurement; eg kilogram, metre, second, litre, metre per second, Celsius scale and be able to convert between different units.
- Perform currency conversions including commission charges
- Use the GDC to solve pairs of linear equations with 2 variables (TI 84 = PLYSMT: simultaneous equations; or graphically: point of intersection)
- Use the GDC to solve quadratic equations (TI 84 = PLYSMT: polyroot finder; or graphically: zeros)
- Apply the formulae for arithmetic sequences and series to find the n th term and the sum of the first n terms of the sequence. Identify the first term U_1 and the common difference d .
- Apply the formulae for geometric sequences and series to find the n th term and the sum of the first n terms of the sequence. Identify the first term U_1 and the common ratio r .
- Calculate compound interest with yearly, semi-annually, quarterly or monthly payments.
- Calculate annual depreciation.

Topic 2: Descriptive Statistics

- Classify data as either discrete or continuous.
- Construct and interpret frequency tables of discrete data
- Construct and interpret frequency tables of grouped discrete or continuous data using intervals.
- Draw histograms from frequency tables
- Calculate cumulative frequency and draw cumulative frequency curves and use these to calculate the median and quartiles
- Draw and interpret box and whisker diagrams
- Calculate the mean, median and mode of discrete data
- Estimate the mean and modal class of grouped discrete and continuous data
- Calculate the range and interquartile range
- Use the GDC to calculate standard deviation (σ_x on GDC)

Topic 3: Logic, Sets and Probability

- Know the definition of a proposition and its symbolic notation
- Define and use compound statements in truth table: conjunction \wedge ; disjunction \vee ; exclusive disjunction $\underline{\vee}$; negation \neg ; implication \Rightarrow ; and equivalence \Leftrightarrow
- Translate between verbal statements and symbolic form (and vice versa)
- Construct truth tables to solve statements between 2 or 3 propositions
- Understand concepts of logical contradiction, logical equivalence and tautology
- Define converse, inverse and contrapositive
- Test the validity of simple arguments using truth tables
- Know the basic concepts of set theory: universal set U , elements $x \in A$, subsets $A \subset B$, intersection $A \cap B$, union $A \cup B$, complement A' , empty set \emptyset
- Draw and interpret Venn diagrams
- Use sample space diagrams to find probability of an event A or a complementary event A'

- Calculate expected value
- Calculate probability of combined events, mutually exclusive events and independent events.
- Draw and interpret tree diagrams, Venn diagrams, sample space diagrams and tables of outcomes to calculate probability
- Calculate probability both “with replacement” and “without replacement”
- Calculate conditional probability

Topic 4: Statistical Applications

- Understand the normal distribution and the distribution of data between the mean μ and $\pm \sigma$ (68%), $\mu \pm 2\sigma$ (95%) and $\mu \pm 3\sigma$ (99%)
- Understand the concepts of a random variable, the parameters μ and σ , the bell shape and the symmetry about $x = \mu$
- Draw a diagrammatic representation of the normal distribution and be able to shade appropriately to show working
- Use GDC to calculate normal probabilities or inverse normal problems
- Calculate expected value
- Understand the concept of correlation with bivariate data and recognize the difference between correlation and causation
- Draw and interpret scatter diagrams including drawing the line of best fit by eye passing through the mean point
- Use the GDC to calculate Pearson’s product-moment correlation coefficient r
- Interpret the r value for positive, zero and negative, strong or weak correlations
- Use the GDC to calculate the regression line for y on x ($y = mx + c$)
- Interpret the regression line to make predictions and understand the dangers of extrapolation
- Use the GDC to solve problems involving χ^2 test for independence
- Formulate the null and alternative hypotheses, understand significance levels and construct contingency tables.
- Calculate expected values and degrees of freedom by hand
- Use the GDC to calculate the χ^2 and p values and then interpret the critical value (given) to accept or reject the null hypothesis

Topic 5: Geometry and Trigonometry

- Calculate the equation of a line and express it in *gradient-intercept* form ($y = mx + c$) and *standard* form ($ax + by + d = 0$)
- Calculate the gradient of a line and find its intercepts (x and y axes)
- Find the points of intersection of lines (graphically and with GDC: simultaneous equations)
- Understand that lines with gradients m_1 and m_2 are parallel when $m_1 = m_2$ and perpendicular when $m_1 \times m_2 = -1$
- Calculate the sides and angles of right-angled triangles using sine, cosine and tangent ratios
- Calculate the angles of elevation and depression
- Use the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to calculate the sides and angles of non-right angled triangles
- Use the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ to calculate sides and $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ to calculate angles of non-right angled triangles
- Calculate the area of non-right angled triangles using $Area = \frac{1}{2}absinC$
- Construct appropriate labeled diagrams from verbal statements
- Calculate angles and sides of three-dimensional geometric solids: cuboid, right prism, right pyramid, right cone, cylinder, sphere, hemisphere and combinations of these solids

- Calculate the distance between two points on a solid; eg between two vertices, or vertices with midpoints or midpoints with midpoints
- Calculate the size of an angle between two lines or between a line and a plane
- Calculate the volume and surface area of the three-dimensional solids listed above

Topic 6: Mathematical Models

- Understand the concept of a function, domain, range and graph
- Understand and use function notation, eg $f(x)$, $v(t)$, $C(n)$,
- Understand the concept of a function as a mathematical model
- Recognise linear functions and draw their graphs eg $f(x) = mx + c$
- Recognise quadratic functions and draw their graphs eg $f(x) = ax^2 + bx + c$; $a \neq 0$
- Know the properties of a parabola: symmetry, vertex, intercepts on the x axis (roots or zeros) and y axis
- Use the equation for the axis of symmetry, $x = \frac{b}{2a}$
- Recognise exponential functions and draw their graphs eg $f(x) = ka^x + c$; $a \in \mathbb{Q}^+$, $a \neq 1$, $k \neq 0$ and $f(x) = ka^{-x} + c$; $a \in \mathbb{Q}^+$, $a \neq 1$, $k \neq 0$
- Understand the concept and write the equation of a horizontal asymptote
- Solve problems using functions in the form $f(x) = ax^m + bx^n + \dots$; $m, n \in \mathbb{Z}$
- Graph functions of this type, eg $f(x) = 3x^4 - 5x + 3$ and $g(x) = 3x^2 - \frac{4}{x}$
- Draw accurate graphs (choosing appropriate scales and plotting points) with labels
- Create a sketch from information given (also with scale indicated and labels)
- Transfer a graph from the GDC to paper (either sketching or using points from table to draw)
- Read, interpret and make predictions using graphs
- Use GDC to solve equations involving combinations of the functions above (points of intersection)

Topic 7: Introduction to Differential Calculus

- Understand the concept of the derivative as a rate of change and its relationship to the tangent of a curve
- Calculate the derivative where $f(x) = ax^n$ and $f'(x) = anx^{n-1}$
- Recognise alternative notations for the derivative, eg $\frac{dx}{dy}$ or $\frac{dV}{dr}$
- Calculate the derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where all exponents are integers (positive and negative)
- Calculate the gradients of curves for given values of x (by hand and with a GDC)
- Deduce the values of x where $f'(x)$ is given
- Deduce the equation of a tangent at a given point (by hand and with a GDC)
- Deduce the equation of a line perpendicular to the tangent at a given point (the normal) (by hand and with a GDC)
- Recognise areas of functions which are increasing or decreasing
- Interpret $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$ and hence sketch a corresponding function $f(x)$
- Find the values of x where the gradient of a curve is zero and solve $f'(x) = 0$
- Understand the concept of stationary points and be able to find the local maximum and minimum points within a stated domain
- Use concepts of stationary points to solve optimization problems.