Arithmetic Sequences and Series

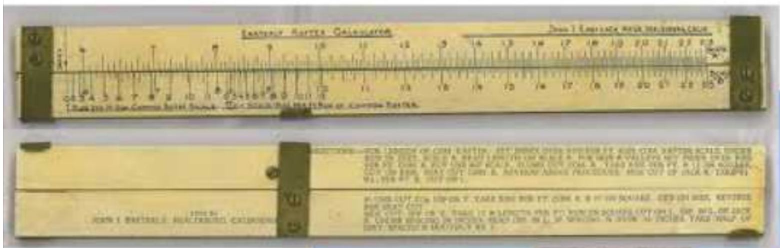
Homework:

Due on Thursday, March 20th.

PART 1: Page 298-7A: ALL ~~Due on Tuesday, 7th of January!~~

PART 2: Page 301-302:-7B: ALL

In the beginning, there were patterns…



Recognizing the pattern…

- 14, 17, 20, 23, … 26, 29, 32… (+3)
- 8, 16, 24, 32, … 40, 48, 56… (+8)
- 36, 31, 26, 21, … 16, 11, 6… (−5)
- 1, 4, 16, 64, …
- 480, 240, 120, 60, … 30, 15, 7.5 (÷2)
- 50000, 10000, 2000, 400, … 80, 15… (÷5)

Arithmetic Sequences

- An Arithmetic Sequence is a sequence of numbers in which each term differs from the previous one by the same fixed constant "d" number. They can be finite or infinite (symbol …) And the elements of it are called terms .

d is also called "common difference"

- Let's investigate some formulas!!!

$u_1, u_2, u_3 \ldots$

$$\boxed{u_3 - u_2 = u_2 - u_1}$$

$$u_3 + u_1 = u_2 + u_2$$

$$2u_2 = u_1 + u_3$$

$$\boxed{u_2 = \frac{u_1 + u_3}{2}}$$

any 3 consecutive

$u_{middle} = \frac{before + after}{2}$

arithmetic average

$u_2 = u_1 + d$

$u_3 = u_2 + d$

$u_4 = u_3 + d$

⋮

General Term Formula

- Let's investigate:

$u_2 = u_1 + d$

$u_3 = u_2 + d$

$u_3 = \overbrace{u_1 + d} + d$

$u_3 = u_1 + 2d$

$u_4 = u_3 + d$

$u_4 = \overbrace{u_1 + 2d} + d$

$u_4 = u_1 + 3d$

$\vdots$

- The formula for

$$u_n = u_1 + (n-1)d$$

Where:

$u_n =$ the "n"th term

$u_1 =$ the 1^{st} term

$n =$ "number of terms"

$d =$ difference (constant)

Examples:

- Given a sequence of numbers: 2, 5, 8 ,11, 14, 17, …
  - a) Show that the sequence is an arithmetic sequence
  - b) Write down the common difference
  - c) Find the 10th term
  - d) Find the 25th term

2 methods to answer part a)

a) $8-5 \stackrel{?}{=} 5-2$ ($u_3 - u_2 = u_2 - u_1$)

$3 = 3$ ✓

$14 - 11 = 11 - 8$

$3 = 3$ ✓

$\therefore d = 3$ a constant

$11 \stackrel{?}{=} \frac{14+8}{2}$

$11 = 11$ ✓

$5 = \frac{2+8}{2}$

$5 = 5$ ✓

b) $d = 3$

c) $u_{10} = u_1 + 9d$

$= 2 + 9(3)$

$\boxed{u_{10} = 29}$

d) $u_{25} = u_1 + 24d$

$= 2 + 24(3)$

$\boxed{u_{25} = 74}$

Examples

- For the sequence 2, 9, 16, 23, 30, … $\Rightarrow d = 9-2 = 7$

a) Find the formula for the general term u_n

b) Hence Find the 100^{th} term of the sequence

c) Is 828 a term of the sequence? Is 2341?

a)

$$u_n = u_1 + (n-1)d$$
$$u_n = 2 + (n-1)7$$
$$u_n = 2 + 7(n-1)$$
$$u_n = 2 + 7n - 7$$
$$\boxed{u_n = 7n - 5}$$

b) 100^{th} term

$$u_{100} = 7(100) - 5 \Rightarrow \boxed{u_{100} = 695}$$

c)

$$828 = 2 + (n-1)(7)$$

or

$$828 = 7n - 5$$
$$7n = 833$$
$$n = 119$$

Yes! it is the 119^{th} term. $u_{119} = 828$

$$2341 = 7n - 5$$
$$7n = 2346$$
$$n = 335.14\ldots$$

NO! because n is not a whole # $n \notin \mathbb{Z}^+$

Examples

- For the sequence of numbers: 6 10 14 … 50 Finite
  a) Write down the common difference
  b) Find the number of terms in the sequence.

a) $d = 10 - 6 = 4$

b) $u_n = 50$
$d = 4$
$u_1 = 6 \quad n?$

$50 = 6 + (n-1)4$
$50 = 6 + 4n - 4$
$50 = 2 + 4n$
$48 = 4n \Rightarrow n = 12$

The sequence has 12 terms

- The second term of an arithmetic sequence is 1 and the seventh term is 26.
  a) Find the first term and the common difference .
  b) Find the 100th term.

a) $u_1 = ? \quad d = ?$
$u_2 = 1$
$u_7 = 26$

$u_7 = u_1 + 6d$
$\Rightarrow 26 = u_1 + 6d$
$u_2 = u_1 + d$
$\Rightarrow 1 = u_1 + d$

$$\begin{cases} 26 = u_1 + 6d \\ 1 = u_1 + d \Rightarrow u_1 = 1 - d \end{cases}$$

Solve from GDC $\begin{cases} x_1 = -4 \\ x_2 = 5 \end{cases}$

$u_1 = -4$
$d = 5$

$26 = (1-d) + 6d$
$26 = 1 + 5d$
$25 = 5d$
$\boxed{d = 5}$

$\Rightarrow u_1 = 1 - 5$
$\boxed{u_1 = -4}$

Examples

- Find k given that $3k+1$, k, and -3 are consecutive terms of an arithmetic sequence.

$$k = \frac{(3k+1)+(-3)}{2}$$

$$2k = 3k+1-3$$

$$2k = 3k-2$$

$$-k = -2$$

$$\boxed{k=2}$$

$$k-(3k+1) = -3-k$$

$$-2k+1 = -3-k$$

$$-k = -2$$

$$\boxed{k=2}$$

- Find the general term u_n for an arithmetic sequence given that $u_3 = 8$ and $u_8 = -17$

(3rd term) $8 = u_1 + 2d$

(8th term) $-17 = u_1 + 7d$

using GDC $u_1 = 18$; $d = -5$

$$u_n = 18 + (n-1)(-5)$$

$$u_n = 18 - 5n + 5$$

$$u_n = 22 - 5n$$

Tricky example from homework:

Page 299-#5: the nth term u = 42-3n

a) first term u1 = 42 - 3(1)

   second term u2 = 42-3(2)

b) -9 = 42-3n, solve for n

c) uk = 42 -3k and   u(k+1) = 42-3(k+1)=42-3k-3=39-3k

NOw, because we now that the sum of

uk + u(k+1) is equal to 33, we can write uk + u(k+1) = 33

Which translates into:

[42-3k] + [39-3k] = 33

81 - 6k = 33

-6k = -48

k = 8