

Arithmetic Series

- An arithmetic Series is the Sum all the successive or consecutive terms of a sequence

eg: $S_n = u_1 + u_2 + u_3 + u_4 + \dots + u_n$

2 Formulas:

$$S_n = \frac{n}{2}(u_1 + u_n)$$

$$S_n = \frac{n}{2}(u_1 + u_1 + (n-1)d)$$

$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

$$u_n = u_1 + (n-1)d$$

Examples

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) \quad // \quad S_n = \frac{n}{2}(u_1 + u_n)$$

- Find the sum of $4 + 7 + 10 + 13 + \dots$ to 50 terms.

$S_{50}?$ $4, 7, 10, 13, \dots$
 $d = 3$

$$S_{50} = \frac{50}{2}(2(4) + (50-1)3)$$

$$S_{50} = 25(8 + 147) = 3875$$

- Find the sum of $-6 + 1 + 8 + 15 + \dots + 141$

Sum? but we don't have "n"
 $d = 7$ $141 = u_1 + (n-1)d$

$$147 = 7n - 7$$

$$154 = 7n$$

$$n = 22$$

$$S_{22} = \frac{22}{2}(-6 + 141)$$

$$S_{22} = 11(135)$$

$$S_{22} = 1485$$

Now that we have $n=22$ we can find S_{22} ?

Examples (application with ^{MONEY} yearly installments)

- Susan wants to buy a flat. She has to pay for the flat in 20 yearly installments. The first installment is 5500 euros. Each installment is 500 Euros more than the one before.

- Write down the values of the second and third installments.
- Calculate the value of the final installment.
- Show that the total amount that Susan would have to pay for the flat is 205 000 Euros.

a) $u_1 = 5500$ $d = 500$
 $u_2 = 6000$
 $u_3 = 6500$

as soon as you see total amount
 ↓
 think Series

b) $u_{20}?$ $u_n = u_1 + (n-1)d$

$u_{20} = 5500 + 19(500)$
 $= 15000$ $S_n = \frac{n}{2}(u_1 + u_n)$

c) $S_{20} = \frac{20}{2}(5500 + 15000) = 205\ 000$

Examples

NO Series here
 just Sequence

- The sales of smartphones are growing every year. At the end of 2006, the number sold was 25 000 000. At the end of 2010, the number sold was 35 800 000. Assuming that the sales figures follow an arithmetic sequence, calculate.
- The number of smartphones sold at the end of 2008 $u_3?$
 - The predicted number of smartphones sold at the end of 2015 $u_{10}?$

$u_1 = 25\ 000\ 000$

$u_5 = 35\ 800\ 000$

$u_3?$ $u_5 = u_1 + 4d$

$35\ 800\ 000 = 25\ 000\ 000 + 4d$
 $d = 2\ 700\ 000$

At the end of 2008 $\Rightarrow u_3 = u_1 + 2d$
 $= 25\ 000\ 000 + 2(2\ 700\ 000)$
 $= 30\ 400\ 000$

Examples

- A woman deposits \$50 into her daughter's savings account on her 1st birthday. On her second birthday she deposits \$75, on her third birthday \$100, and so on. $d = 25$
- a) How much money will she deposit in her daughter's account on her 18th birthday? $u_{18}?$
- b) How much money in total will she have deposited after her daughter's 18th birthday? $S_{18}?$

a) $u_{18} = u_1 + 17d$
 $u_{18} = 50 + 17(25)$
 $u_{18} = 475$

b) $S_{18} = \frac{18}{2}(50 + 475)$
 $S_{18} = 4725$

Examples

- Zain goes swimming. He swims the first length of the pool in 2.5 minutes. The time he takes to swim each length is 10 seconds more than he took to swim the previous length. $u_1 = 150s$
 $u_1 = 2.5min$
- a) Find the time Zain takes to swim the 3rd length. $u_3?$
- b) Find the time taken for Zain to swim a total of 10 lengths of the pool. $S_{10}?$

a) $u_3 = 150 + 2(10)$
 $u_3 = 170 \text{ seconds}$

b) $S_{10} = \frac{10}{2}(2(150) + 9(10))$
 $S_{10} = 1950 \text{ seconds}$

Examples

- Mr. Zheng decides to increase the amount of money he gives to charities by p yen every year. In the first year he gives a yen. In the 6th year he gives twice as much as in the 3rd year. In the 10th year he gives 4000 yen. Find the value of p and a .

$$\begin{aligned}
 & \left. \begin{aligned} u_1 &= a \\ d &= p \\ u_6 &= 2u_3 \\ u_{10} &= 4000 \end{aligned} \right\} \begin{aligned} & \rightarrow 4000 = a + (10-1)p \\ & 4000 = a + 9p \\ & \text{1st equation} \end{aligned} \\
 & \left. \begin{aligned} u_6 &= 2u_3 \\ u_{10} &= 4000 \end{aligned} \right\} \begin{aligned} & u_3 = a + 2p \\ & \text{2nd equation} \end{aligned} \\
 & \left. \begin{aligned} u_6 &= 2u_3 \\ u_{10} &= 4000 \end{aligned} \right\} \begin{aligned} & \rightarrow 2u_3 = a + 5p \\ & (\text{or } u_6) \text{ 3rd equation} \end{aligned}
 \end{aligned}$$

This is a bit trickier because now you are solving a system of 3 equations and you are not so used to doing so. It is not that hard, you can solve by substitution. But better yet....solve with your GDC! do you know how?

First, I will solve by hand...

$$\begin{aligned}
 & \text{① } 4000 = a + 9p \\
 & \text{② } u_3 = a + 2p \\
 & \text{③ } 2u_3 = a + 5p
 \end{aligned}$$

From equation ① $a = 4000 - 9p$
 Now substitute eq ① into ② & ③

$$\begin{aligned}
 & u_3 = (4000 - 9p) + 2p \\
 & 2u_3 = (4000 - 9p) + 5p
 \end{aligned}$$

New system

$$\begin{cases} u_3 = 4000 - 7p \\ 2u_3 = 4000 - 4p \end{cases}$$

Now rearrange and simplify new system

$$\begin{aligned}
 & \text{Solve by elimination} \\
 & \begin{cases} (u_3 = 4000 - 7p) \times (-2) \\ 2u_3 = 4000 - 4p \end{cases} \\
 & \begin{cases} -2u_3 = -8000 + 14p \\ 2u_3 = 4000 - 4p \end{cases} \\
 & \hline
 & \quad \quad \quad = -4000 + 10p \\
 & \quad \quad \quad 4000 = 10p \\
 & \quad \quad \quad p = 400
 \end{aligned}$$

Now from eq ① $a = 4000 - 9p$
We can substitute $p = \frac{4000}{3}$ into ①

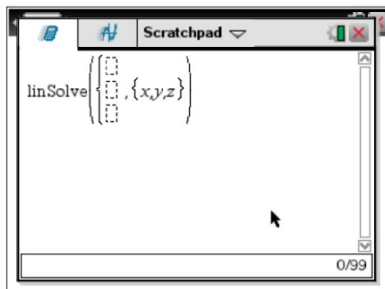
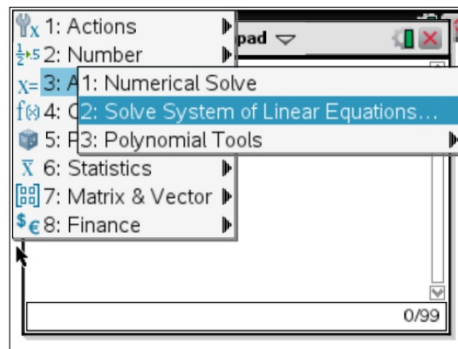
$$a = 4000 - 9(400)$$

$$a = 4000 - 3600$$

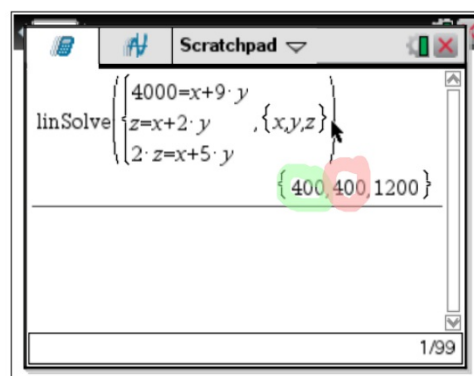
$$a = 400$$

Secondly, solve with N-Spire:

1. on Scratchpad, go to menu
2. go to 3:Algebra
3. go to 2:solve System of Linear



4. now you see the screen t the left. Enter the 3 equations as given
5. move cursor to the end of the line and hit ENTER.



Now like magic, here are your answers.

I entered a as an x

I entered p as a y

I entered u_3 as the z

Therefore,

$$a = 400$$

$$p = 400$$

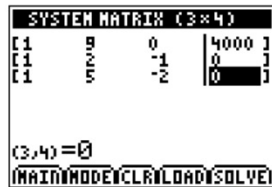
$u_3 = 1200$ (but we do not need to solve for u_3)

3rd Solve with TI-84:

Original equations: 1. Go to "plymsmlt2" under apps
4000 = a + 9y 2. go to 2: SIMULT EQN SOLVER
z = a + 2y
2z = a + 5y

5. you must rearrange system before you can enter coefficients.

$$\begin{aligned} a + 9p &= 4000 \\ a + 2p - u_3 &= 0 \\ a + 5p - u_3 &= 0 \end{aligned}$$



6. Hit solve (graph)

Here are your solutions:

x_1 is what we call a

x_2 is what we called p

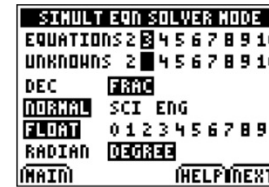
x_3 is what we called u_3

Hence,

$$a = 400$$

$$p = 400$$

$$u_3 = 1200 \text{ (but we do not need it)}$$



3. Make sure you have 3 equations and 3 unknowns.

4. Hit next...(graph)

