Geometry 3-Non Right Δs

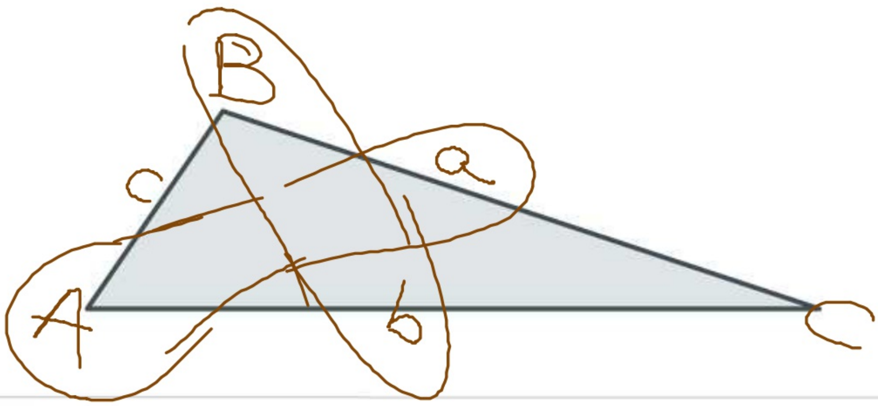
Homework:
Page 121-3N: odd + #8
Page 123-3O: odd + #8
Page 125-3P: 3 to 7

Law of Sines

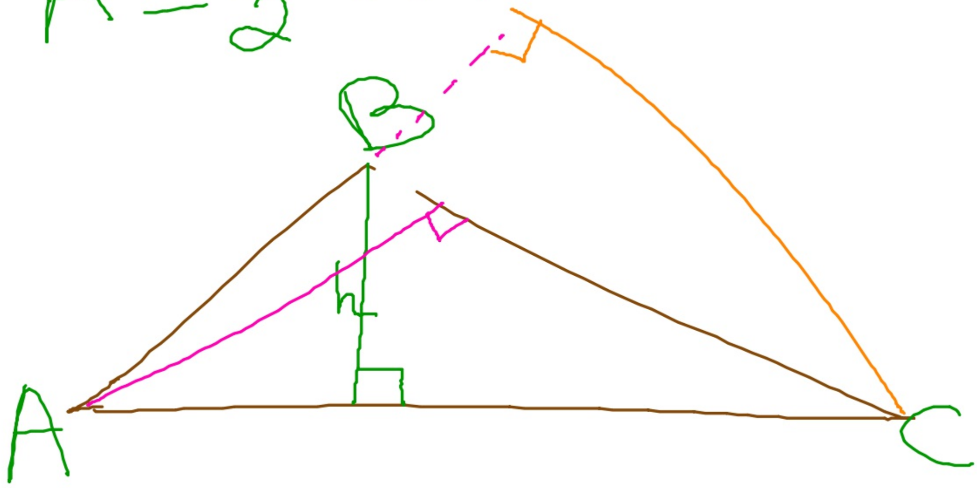
Given a ΔABC, with side lengths a, b, and c; then we know that

If this is True, then this is also True

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \qquad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



$$A = \frac{1}{2} b \times h$$



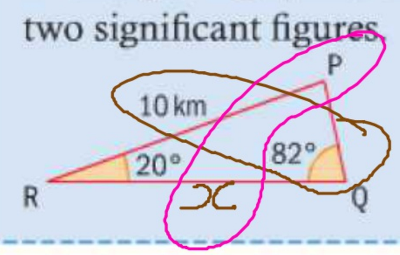
If you are looking for the area of ΔABC, you can use any of the 3 Heights, but then your base changes depending on the height. Remember base (little b) and height (little h) are ALWAYS at a 90˚ angle.

- If you use the green height, then the base is AC
- If you use the pink height, then the base is BC
- If you use the orange height, then the base is AB

Obviously, they easiest to visualize is the green height and base AC.

Examples

In triangle PQR, find the length of RQ. Give your answer correct to two significant figures.



$$m\angle P = 180 - (20 + 82)$$
$$= 78^\circ$$

$$\frac{x}{\sin 78^\circ} = \frac{10}{\sin 82^\circ}$$

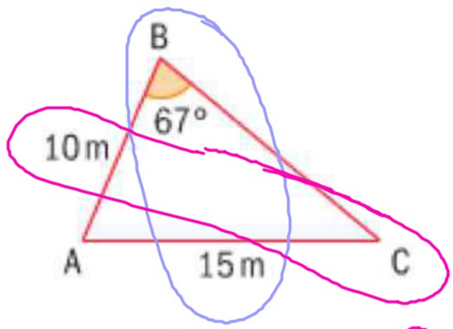
$$x = \frac{10 \sin 78^\circ}{\sin 82^\circ} = 9.87760\ldots$$

$$\boxed{RQ = 9.9 \text{ km}}$$

Examples

In each triangle, find the angle indicated.

a $\hat{C}$



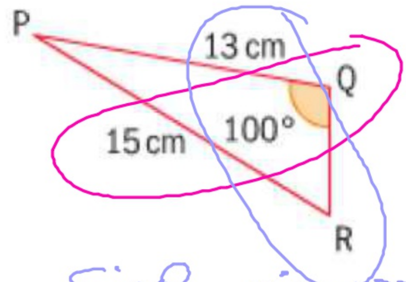
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{10} = \frac{\sin 67°}{15}$$

$$\sin C = \frac{10 \sin 67}{15}$$

$$C = \sin^{-1}\left(\frac{\quad}{15}\right)$$

b $\hat{R}$



$$\frac{\sin R}{13} = \frac{\sin 100°}{15}$$

$$\sin R = \frac{13 \sin 100}{15}$$

$$R = \sin^{-1}\left(\frac{13 \sin 10}{15}\right)$$

Note: when entering in GDC, either enter the whole expression (13sin10)/15 or enter (Ans) so that you carry all the decimals...DO NOT ROUND TILL THE END!

Law of Cosines (finding sides)

REMEMBER: YOU ARE ONLY GETTING THE TOP FORMULA ON THE TEST.

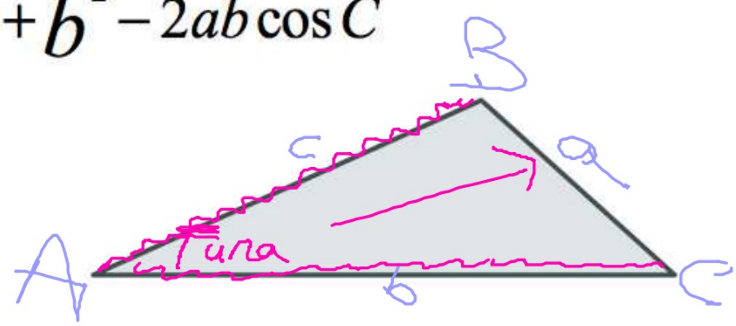
Given a ΔABC with side lengths a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

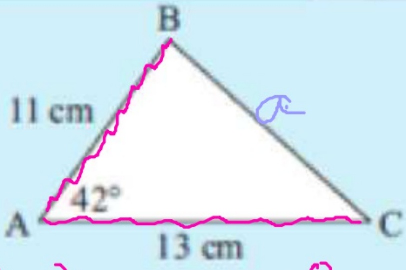
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Where your angle A is the inside of the sandwich (tuna), and b and c stand for the slides of bread of your sandwich.



Examples $a^2 = b^2 + c^2 - 2bc\cos A$

Find, correct to 2 decimal places, the length of BC.



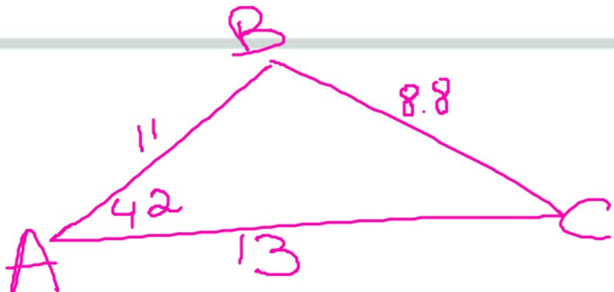
$$a^2 = 11^2 + 13^2 - 2(11)(13)\cos 42^\circ$$

$$a^2 = 121 + 169 - 212.539420\ldots$$

$$a = 8.801169\ldots \text{ or } \boxed{8.80\text{ cm}}$$

The most difficult part is not the "geometry" of knowing how to use Cosine Law, but to enter all this correctly in calculator.
Be careful.

Cosine Law (finding angles)



Now solve for angle C, using Cosine Law and rearrange.

$$11^2 = 13^2 + 8.8^2 - 2(13)(8.8)\cos C$$

$2(13)(8.8)\cos C = 13^2 + 8.8^2 - 11^2$

$$\cos C = \frac{13^2 + 8.8^2 - 11^2}{2(13)(8.8)}$$

$\cos C = 0.5482517\ldots$

$\angle C = \cos^{-1}(0.5482517\ldots)$ don not round, use "ctrl" "ans"

$\angle C = 56.8^\circ$

As you can see, it can get messy (ugly algebra)==> But it has been done for you....SEE NEXT SLIDE

Cosine Law (finding angles)

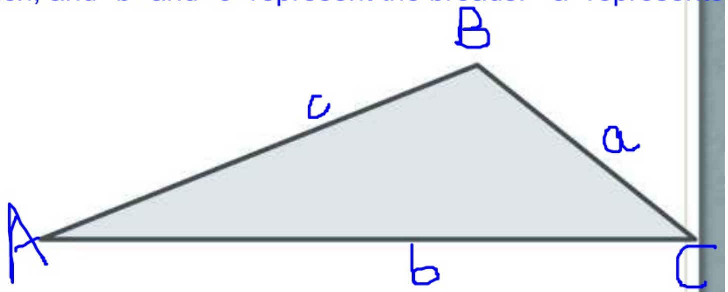
REMEMBER: YOU ONLY GET THE TOP FORMULA IN EXAM

Again, the angle is the inside (tuna) of the sandwich, and "b" and "c" represent the breads. "a" represents the side opposite of the "tuna".

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

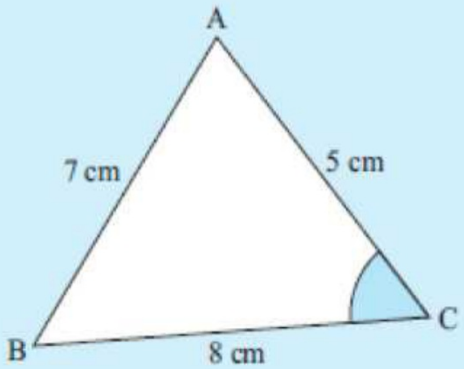
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Example

In triangle ABC, if AB = 7 cm, BC = 8 cm and CA = 5 cm, find the measure of angle BCA.



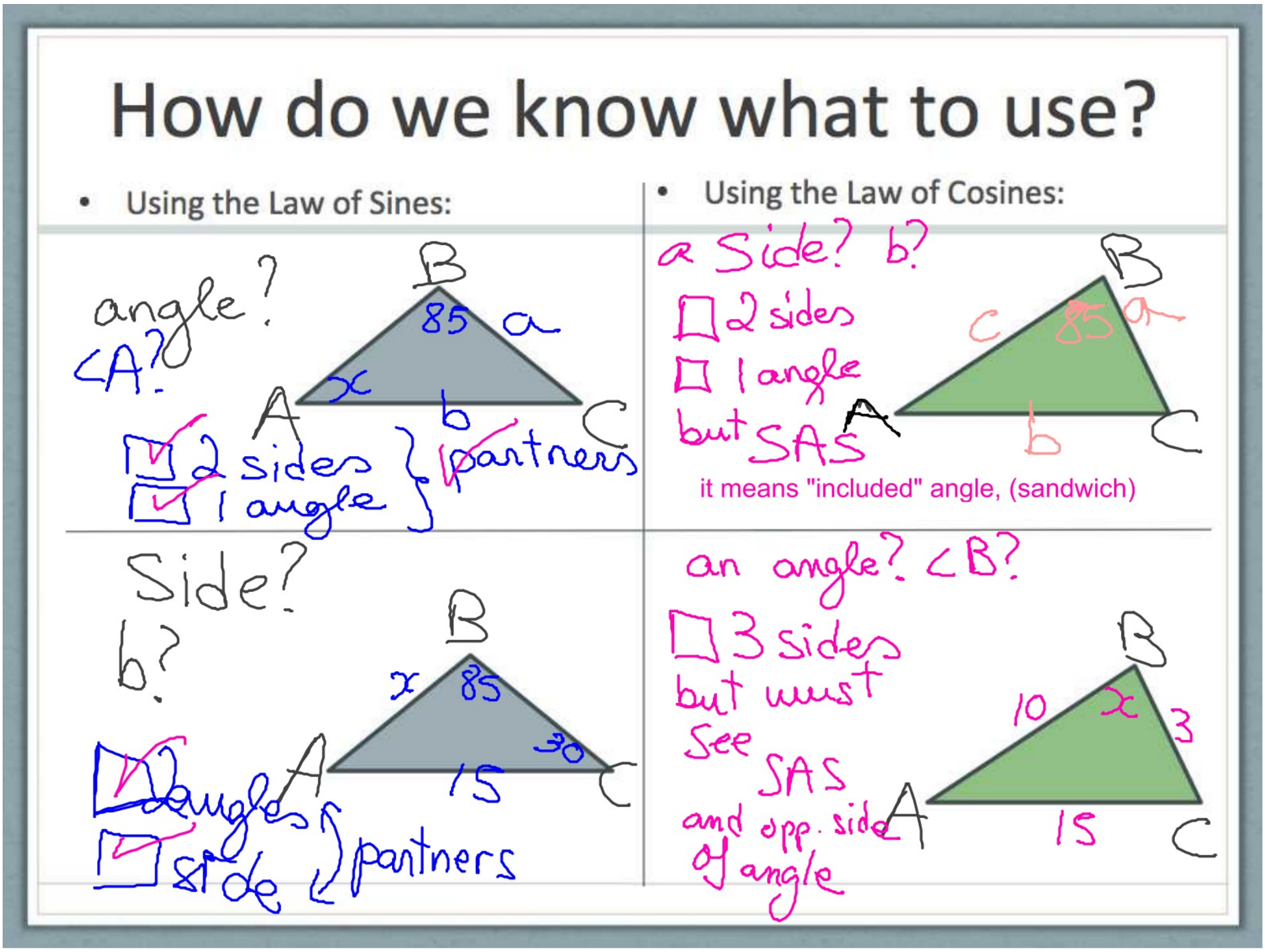
By the cosine rule:

$$\cos C = \frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}$$

$$\therefore \quad C = \cos^{-1}\left(\frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}\right)$$

$$\therefore \quad C = 60$$

So, angle BCA measures 60^o.

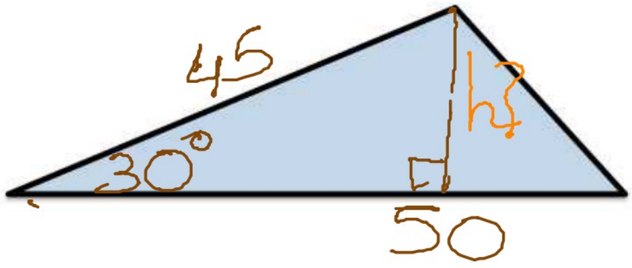


Application of Non-Right Δ TRIG: Angle of Sight

Area Law

Given ΔABC, then the opposite sides will be **a, b, c** respectively. Find the area of ΔABC

Area $= \frac{1}{2} b \times h$



$h = 45 \sin 30$

$A = \frac{1}{2} b \times 45 \sin 30$

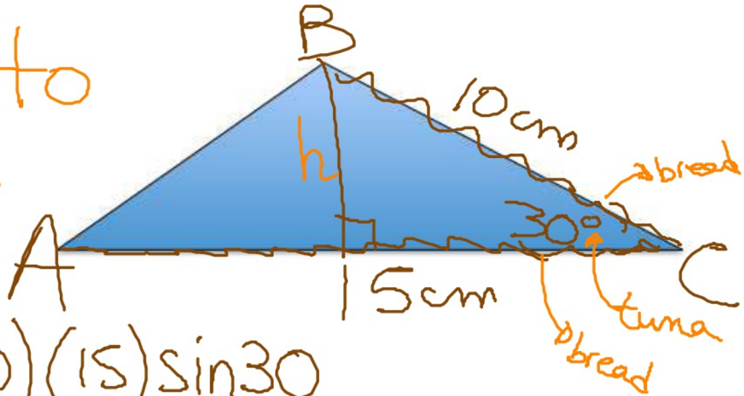
you can always workout the height of the Δ yourself, but the area law has done this for you. SEE NEXT SLIDE

Area Law

$$A = \frac{1}{2} \underbrace{ab}_{\text{breads}} \underbrace{\sin C}_{\text{(tuna)}}$$

Given ΔABC, where BC=10cm and AC=15cm and <C measures 30°. Find the its area.

No need to Solve for h



Area $= \frac{1}{2}(10)(15)\sin 30$

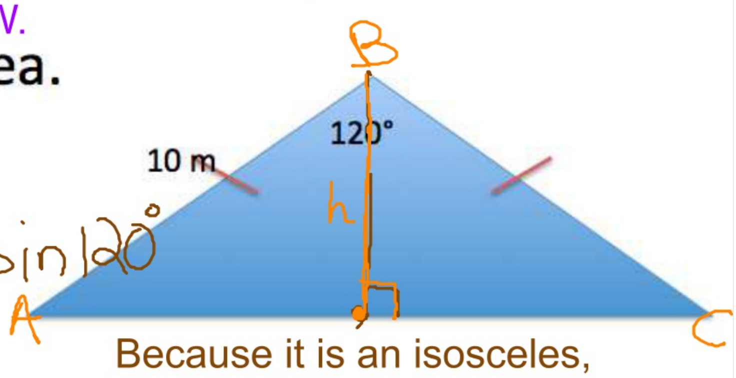
Area $= 150\ cm^2$

Area Law $A = \frac{1}{2}ab\sin C$

Because it is an isosceles Δ, there are many ways of solving this area. However, the fastest is AREA LAW.

Tricky one!!!

Given ΔQPT, find its area.



Because it is an isosceles, the height (h) happens to be the angle bisector of 120° and the perpendicular bisector of AC.

$$\text{Area} = \frac{1}{2}(10)(10)\sin 120^\circ$$

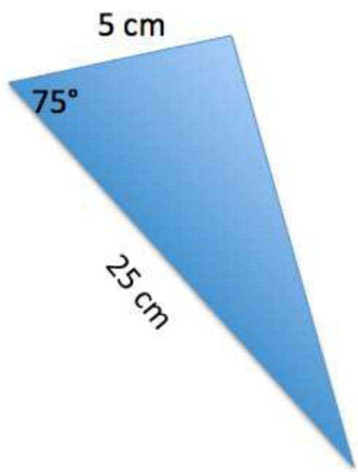
$$\text{Area} = \frac{100}{2}\sin 120$$

$$= 50\sin 120$$

$$= 43.3\ m^2$$

Area Law $A = \frac{1}{2}ab\sin C$

- Even more tricky! Given ΔABC, find its area



$$\text{Area} = \frac{1}{2}(5)(25)\sin 75^\circ$$

$$= 60.3703\ldots$$

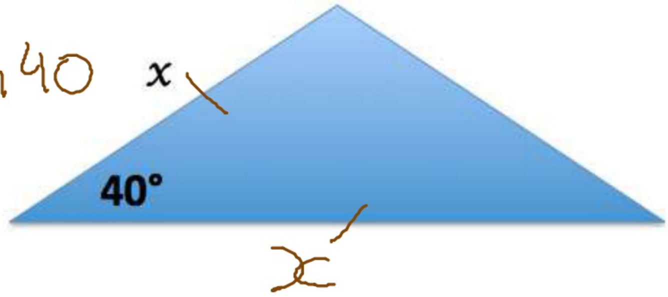
$$\boxed{= 60.4\ cm^2}$$

Application of Area Law

$$A = \frac{1}{2}ab\sin C$$

- Given isosceles ΔABC, with an area of 200 cm², find x.

$$\text{Area} = \frac{1}{2}(x)(x)\sin 40$$



$$200 = \frac{1}{2}x^2\sin 40$$

$$\frac{400}{\sin 40} = x^2$$

$$x = \sqrt{\frac{400}{\sin 40}} = 24.945\ldots$$

$$= 25.0$$