Review Package for Quadratic Functions—Due after winter break

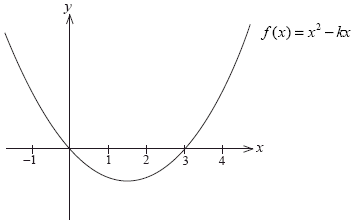
**1.** (a) Factorise the expression *x*2– *kx*.

(1)

(b) Hence solve the equation *x*2– *kx* = 0.

(1)

The diagram below shows the graph of the function *f*(*x*) = *x*2 – *kx* for a particular value of *k*.



(c) Write down the value of *k* for this function.

(1)

(d) Find the minimum value of the function *y* = *f*(*x*).

(3)

(Total 6 marks)

**2.**

The function *Q* (*t*) = 0.003*t*2 – 0.625*t* + 25 represents the amount of energy in a battery after *t* minutes of use.

(a) State the amount of energy held by the battery immediately before it was used.

(b) Calculate the amount of energy available after 20 minutes.

(c) Given that *Q* (10) = 19.05, find the average amount of energy produced per minute for the interval 10  *t*  20.

(d) Calculate the number of minutes it takes for the energy to reach zero.

(Total 6 marks)

**3.**

The diagram below shows the graph of *y* = *c* + *kx* – *x*2, where *k* and *c* are constants.



(a) Find the values of *k* and *c*.

(b) Find the coordinates of Q, the highest point on the graph.

(Total 8 marks)

4.

The graph of the function *y* = *x*2 – *x* – 2 is drawn below.



(a) Write down the coordinates of the point C.

(b) Calculate the coordinates of the points A and B.

(Total 8 marks)

5.

The diagram below shows part of the graph of *y* = *ax*2 + 4*x* – 3. The line *x* = 2 is the axis of symmetry. M and N are points on the curve, as shown.



(a) Find the value of *a.*

(b) Find the coordinates of

(i) M;

(ii) N.

(Total 4 marks)

6.

(a) Solve the equation *x*2 – 5*x* + 6 = 0.

(b) Find the coordinates of the points where the graph of *y* = *x*2 – 5x + 6 intersects the *x*-axis.

(Total 4 marks)

**7.** The diagram shows the graph of *y* = *x*2 – 2*x* – 8. The graph crosses the *x*-axis at the point A, and has a vertex at B.



(a) Factorize *x*2 – 2*x* – 8.

(b) Write down the coordinates of each of these points

(i) A;

(ii) B.

(Total 4 marks)

**8.** (a) Represent the function *y* = 2*x*2 – 5, where *x*  {–2, –1, 0, 1, 2, 3} by a mapping diagram.



(b) List the elements of the domain of this function.

(c) List the elements of the range of this function.

(Total 6 marks)

**9.** A small manufacturing company makes and sells *x* machines each month. The monthly cost *C*, in dollars, of making *x* machines is given by

*C*(*x*) = 2600 + 0.4*x*2.

The monthly income *I*, in dollars, obtained by selling *x* machines is given by

*I*(*x*) = 150*x* – 0.6*x*2.

(a) Show that the company’s monthly profit can be calculated using the quadratic function

*P*(*x*) = – *x*2 + 150*x* – 2600.

(2)

(b) The maximum profit occurs at the vertex of the function *P*(*x*). How many machines should be made and sold each month for a maximum profit?

(2)

(c) If the company does maximize profit, what is the selling price of each machine?

(4)

(d) Given that *P*(*x*) = (*x* – 20) (130 – *x*), find the smallest number of machines the company must make and sell each month in order to make **positive** profit.

(4)

(Total 12 marks)

**10**. The graph of *y* = *x*2 – 2*x* – 3 is shown on the axes below.



(a) Draw the graph of *y* = 5 on the same axes.

(b) Use your graph to find:

(i) the values of *x* when *x*2 – 2*x* – 3 = 5

(ii) the value of *x* that gives the minimum value of *x*2 – 2*x* – 3

(Total 4 marks)

**11.** profit (*P*) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

*P* = –  + 5*x* – 30

where *x* is the number of glasses of lemonade sold.

(a) **Copy** and complete the table below

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| *P* |  | 15 |  |  | 90 |  |  | 75 | 50 |  |

(3)

(b) On graph paper draw axes for *x* and *P*, placing *x* on the horizontal axis and *P* on the vertical axis. Use suitable scales. Draw the graph of *P* against *x* by plotting the points. Label your graph.

(5)

(c) **Use your graph** to find

(i) the maximum possible profit;

(1)

(ii) the number of glasses that need to be sold to make the maximum profit;

(1)

(iii) the number of glasses that need to be sold to make a profit of 80 Swiss Francs;

(2)

(iv) the amount of money initially invested by the three students.

(1)

(d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona’s share of the profits.

(2)

(Total 15 marks)

Answer key to review package

**1.**

(a) *x*(*x* – *k*) (A1) (C1)

(b) *x* **=** 0 **or** *x* **=** *k* (A1) (C1)

**Note:** Both correct answers only

(c) *k* **=** 3 (A1) (C1)

(d) Vertex at *x* =  (M1)

**Note:** (M1) for correct substitution in formula

*x* **=** 1.5 (A1)(ft)  
*y* **=** –2.25 (A1)(ft)

***OR***

*f*′(*x*) **=** 2*x* – 3 (M1)

**Note:** (M1) for correct differentiation

*x* **=** 1.5 (A1)(ft)  
*y* **=** –2.25 (A1)(ft)

***OR***

for finding the midpoint of their 0 and 3 (M1)  
*x* **=** 1.5 (A1)(ft)  
*y* **=** –2.25 (A1)(ft) (C3)

**Note:** If final answer is given as (1.5, –2.25) award a maximum of (M1)(A1)(A0)

[6]

**2.**

(a) *Q* (0) = 25 (A1) (C1)

(b) *Q*(20) = 0.003(20)2 – 0.625(20) + 25 = 13.7 (A1) (C1)

(c) *Q*(20) = 13.7 units and *Q*(10) = 19.05

difference = 19.05 – 13.7 (M1)

*(allow* 13.7 – 19.05*)*

= 5.35

average =  = 0.535 units per minute (sign must be positive) (A1)(ft)

**Note:** The wording may have confused some candidates.  
Hence allow  = 16.375 (16.4) (M1)(A1)(C2) (C2)

and for answer 16.3 (which is Q(15)) award just (C1).  
If the interval is divided into multiple parts (eg 10) then  
the value averaged over these, award no marks. (M1)

(d) 0.003*t2* – 0.625*t* + 25 = 0 (A1) (C2)

**Note:** A sketch showing the first root can earn the (M1).

Energy runs out after 54.0 minutes *(accept 54 but no mark for 154)*

**Note:** 53.99 receives (A0)(AP), 53 is wrong, for 53 – 54 award (A1)

[6]

**3.**

(a) *y* = *x*(5 – *x*) or *y* = 5*x* – *x*2 or 25 = *c* + 5*k* (M1)  
*c* = 0, *k* = 5 (A1)(A1) (C3)

**Note:** Award (A1) if no method is indicated but c = 0 or k = 5 is given alone.

(b) Vertex at *x* = = 2.5 (M1)(A1)  
*y* = 5(2.5) – 2.52 = 6.25 (M1)(A1)

**Note:** The substitutions must be attempted to receive the method marks.

Q(2.5, 6.25) (A1) (C5)

**Notes:** Coordinate pair is required for (A1) but Q is not essential. If no working shown and answer not fully correct, award (G2) for each correct value and (A1) for coordinate brackets. However, if values are close but not exactly correct (eg (2.49, 6.25)) award only (G1) for each less precise value. In this case AP might also apply if number of digits is inappropriate.  
If differentiation is used, award (M1) for correct process, (A1) for x = 2.5, (M1)(A1) or (G2) for 6.25 and (A1) for coordinate brackets.

[8]

**4.**

(a) Put *x* = 0 to find *y* = –2 (M1)  
Coordinates are (0, –2) (A1) (C2)

**Note:** Award (M1)(A0) for –2 if working is shown. If not, award (M0)(A0).

(b) Factorise fully, y = (*x* – 2) (*x* + 1). (A1)(A1)  
*y* = 0 when *x* = –1, 2. (A1)(A1)  
Coordinates are A(–1, 0), B(2, 0). (A1)(A1) (C6)

**Note:** Award (C2) for each correct x value if no method shown and full coordinates not given. If the quadratic formula is used correctly award (M1)(A1)(A1)(A1)(A1)(A1). If the formula is incorrect award only the last (A1)(A1) as ft.

[8]

5.

(a) *x* =   
2 =  (M1)  
*a* = –1 (A1)

(b) ***Note:*** *Answers to (b) must be written as coordinates.*

(i) M(0, –3) (A1)

(ii) *y* = 1 × 22 + 4 × 2 – 3  
 = 1  
N is (2,1) (A1)

[4]

6.

(a) *x*2 – 5*x* + 6 = 0  
(*x* – 2)(*x* – 3) = 0 (A1)  
*x* = 2 (A1)  
*x* = 3 (A1)

(b) (2, 0)  
(3, 0) (A1)

**Notes:** Follow through from part (a). Both must be correct and written as coordinates for (A1)

[4]

**7.** (a) (*x* + 2)(*x* – 4) (A1)

(b) (i) (–2, 0) (A1)

(ii) (1, –9) (A1)(A1)

[4]

**8.** (a)



For six single lines going to correct *y* (*y*-value can be repeated) (M1)  
Correct diagram (*y*-values not repeated) (A1) (C2)

(b) *x*  {–2, –1, 0, 1, 2, 3} (A2) (C2)

**Note:** Award (A1) if one value omitted.

(c) *y*  {–3, –5, 3, 13} (A2) (C2)

[6]

**9.** (a) Profit = Income – Cost  
*P*(*x*) = 150*x* – 0.6*x*2 – (2600 + 0.4*x*2) (M1)  
= 150 *x* – 0.6*x*2 – 2600 – 0.4*x*2(M1)

**Note:** Award (M2) for either line seen without the other, but award only (M1) if omission of brackets results in + 0.4x2.

= –*x*2 + 150*x* – 2600 (AG) 2

(b) maximum profit when *x* =  or *x* =  (M1)  
= 75 machines (A1)  
 or (G2) 2

**Note:** Sketch or table of values from GDC can receive (M1) as long as the values are appropriate. Table must include at least evaluation for 74, 75, 76, and sketch must show 75 beneath the maximum, however, any non-integer answer must receive (A0).  
If differentiation is used, award (M1) for –2x + 150 = 0.

(c) *I* (75) = 150(75) – 0.6(75)2 (M1)  
= $7875 (A1)  
Selling price per machine =  (M1)  
= $105 (A1) 4

**Notes:** If P(75) or C(75) used, award M0 A0 but ft with candidate’s value to the selling price.

(d) *P*(*x*) = 0 or (*x* – 20)(130 – *x*) = 0. (M1)  
*x* = 20 (130 need not be mentioned) (A2)  
Smallest number must be 21. (A1) 4

**Notes:** If no working shown:  
Award (G2) if answer is 20, x > 20 or x = 20,  
Award (G3) if answer is 21 or x = 21.  
A sketch of the function showing the intercepts receives (M1) with (A2) or (A3) for **separate** indication of answer 20 or 21 respectively. If brackets are expanded and quadratic formula is used, the (M1) should be awarded only for correct expansion and correct substitution into the formula.

[12]

**10.** (a)

 (A1) (C1)

**Note:** The equation y = 5 is not required

(b) (i) *x* = –2 (A1)  
*x* = 4 (A1) (C2)

(ii) *x* = 1 (A1) (C1)

**Note:** Allow follow through from candidate’s graph

**11.** (a) (A3)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| *P* | **–30** | 15 | **50** | 75 | 90 | **95** | **90** | 75 | 50 | **15** |

**Note:** Award ½-mark for each correct bold entry, and round down.

If a candidate obtains (A0) here but has clearly shown the method of substituting in the values of x into the formula award (M1)

(b)

 (A2)(A2)(A1)

**Note:** For graph, follow through from candidate’s table

**Notes**: Award (A2) for axes, (A2) for plotting points and (A1) for a smooth curve.

Axes: Award ½-mark for each of the following and then round down:  
horizontal axis labelled with “x” or “Numbers of glasses...”  
vertical axis labelled with “P” of “Profit”  
horizontal scale  consistent and presents values 090  
vertical scale as for horizontal but represents their range of values for P.

Points: Award (A2) for 0 or 1 error  
Award (A1) for 2 or 3 errors  
Award (A0) otherwise

(c) (i) maximum profit = 95 swiss francs (A1)

(ii) 50 glasses (A1)

(iii) 67  2 (A1)  
33  2 (A1)

(iv) 30 swiss francs (A1)

**Note:** Award no marks for –30 swiss francs

**Note:** Follow through from candidate’s graph

(d) Fiona’s share =  (M1)  
Profit from 40 glasses = 90 swiss francs  
Fiona’s profit = ×90  
 = 45 (A1)

[15]