More Functions

Homework:

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Polynomial Functions

$f(x) = ax^n + bx^{n-1} + cx^{n-2} \cdots + d$

Name	*General form*
Polynomial:	
• Linear	$f(x) = ax + b, \quad a \neq 0$
• Quadratic	$f(x) = ax^2 + bx + c, \quad a \neq 0$
• Cubic	$f(x) = ax^3 + bx^2 + cx + d, \quad a \neq 0$
• Quartic	$f(x) = ax^4 + bx^3 + cx^2 + dx + e, \quad a \neq 0$

What could they ask you in an IB question

a) Sketch using your GDC Make sure you know how to adjust window and zoom!

b) Find coordinates of vertices use the maximum and minimum command in GDC!

c) Find the roots or zeros (if any), which they are the x-intercepts

d) y-intercepts Make the x=0, hence y-int. will always be the independent term.

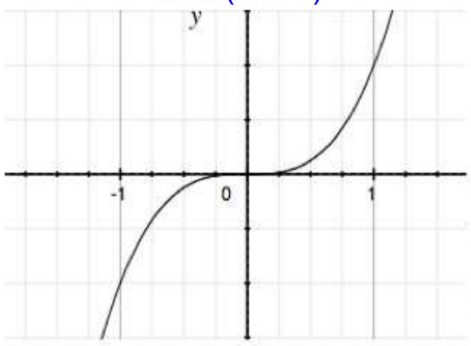
e) domain and Range

- Domain for Polynomial functions is always R, unless "stop signs" are given.
- Range will depend on the graph. So, most likely a sketch will be given.

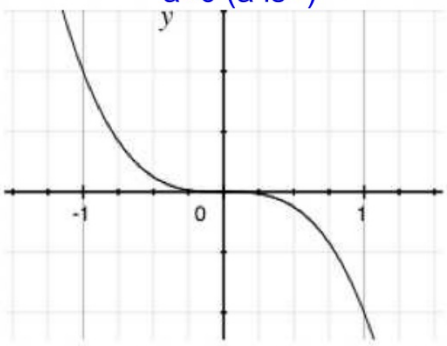
Cubic Polynomials-Use GDC!

$f(x) = ax^3 + bx^2 + cx + d$

a>0 (a is +)



a<0 (a is -)



These are called local max. and locak min. You use the same method in your GDC to get the coordinates of those points as you did with parabolas.

$f(x) = 3x^3 + 4x^2 + 5x$

a) y-int ($x=0$) $\Rightarrow (0,0)$

b) x-int

(Make the y=0) $0 = 3x^3 + 4x^2 + 5x$

Then use your polynomial tools to solve equation.
In this case there is only one solution, hence only ONE x-int.
(x=0) $\Rightarrow (0,0)$

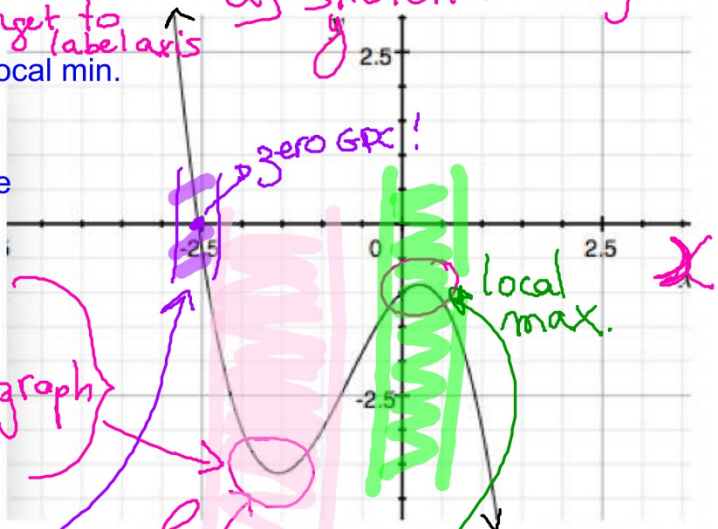
Note: for the x-intercepts, you can also use the command "zeros" from your GDC graphing menu. It is under 6: analyze graph.

Cubic Polynomials (given the equation of the function)

Using your GDC!

a) sketch do not forget to label axis
b) find local max. or local min.
c) Find x-intercepts
d) Find y-intercept
e) Domain and Range

a) sketch 1st in your GDC!



b) use menu 6: analyze graph 2: minimum

Again:
menu
6: analyze graph
3:maximum

c) menu
6: analyze graph
1: zero

d) plug x=0, or just look at independent term.

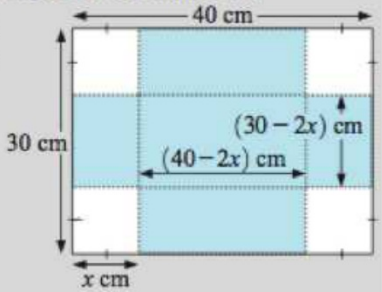
e) domain ==> R or $(-\infty,\infty)$ (same for all polynomial functions and you see it in the sketch)

range ===> R or $(-\infty,\infty)$ (you see it in the sketch)

Application of cubic function or cubic modelling

A 40 cm by 30 cm sheet of tinplate is to be used to make a cake tin.

Squares are cut from its corners and the metal is then folded upwards along the dashed lines.



Edges are fixed together to form the open rectangular tin.

The depth of the tin is x cm and its length and width are $(40 - 2x)$ cm and $(30 - 2x)$ cm respectively.

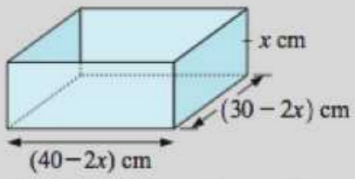
Consequently the capacity of the cake tin V, is given by

$$V(x) = x(40 - 2x)(30 - 2x) \text{ cm}^3$$

or $$V(x) = 4x(20 - x)(15 - x) \text{ cm}^3$$

If expanded, $$V(x) = 4x(300 - 35x + x^2)$$

i.e., $$V(x) = 4x^3 - 140x^2 + 1200x$$



For you to consider:
- a) How does the capacity of the tin change as x changes?
- b) What are the restrictions on the x values, if any?
- c) What sized squares must be cut out for the cake tin to have maximum capacity?

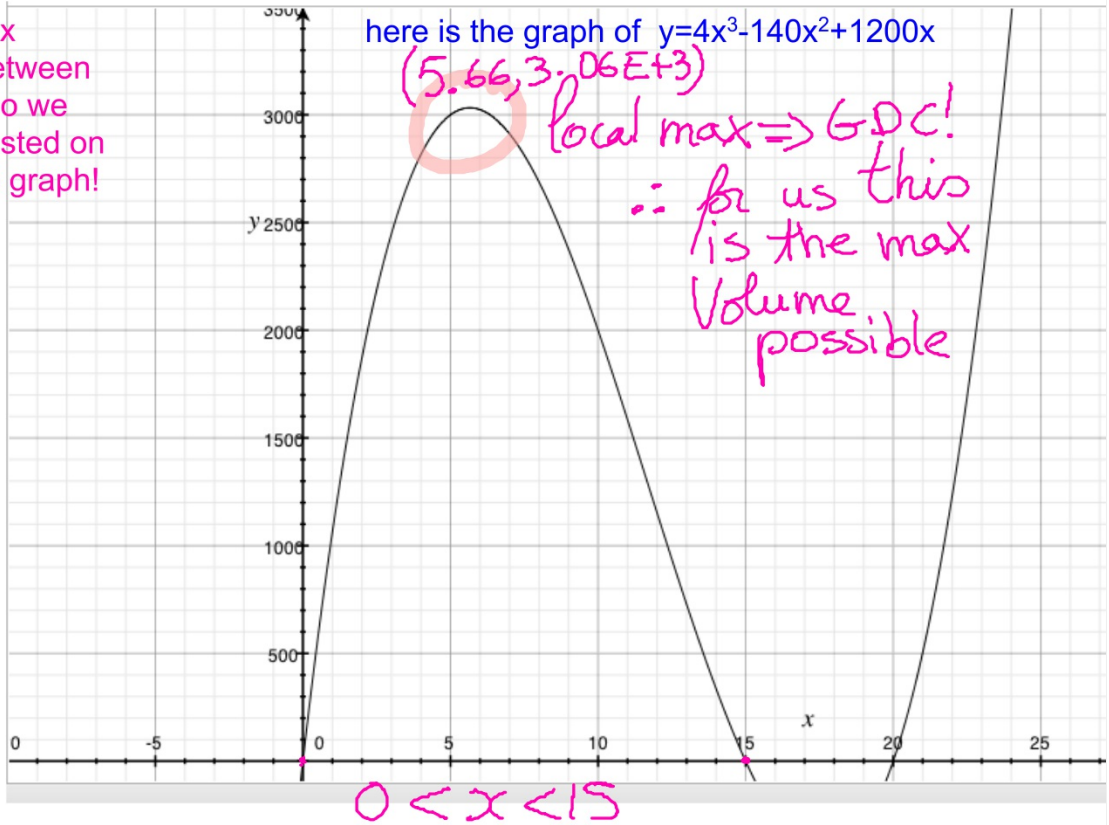
a) it depends on the domain

b) You need to see that $x \neq 0$
(how would you have a box without a height!)

But you also need to see that the width of the box is $30-2x$
Therefore, x must be less than 15 as $30-2(15)=0$ which would mean there is NO BOX!!

Answer: ~~Possible~~ Domain: $0<x<15$

We know that x can only be between zero and 15, so we are only interested on this part of the graph!



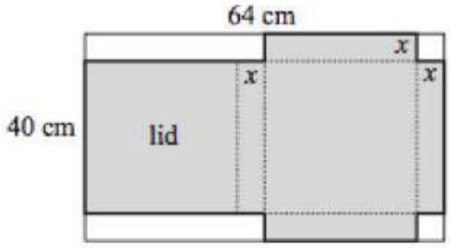
$0<x<15$

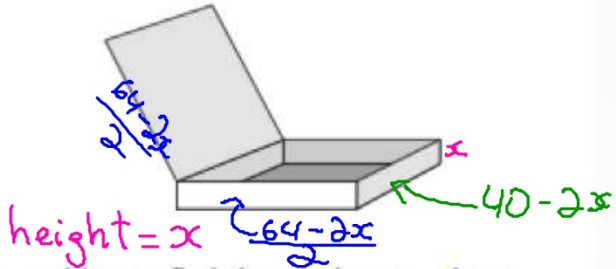
c) Use GDC ⇒ menu: 6: analyze graph; 3: maximum (highlight lower and upper boundary) and you get local max. (5.66, 3.03E+3)

this means that at a height (x) of 5.66 cm (not the maximum height) we obtain the max volume possible 3.03×10^3 cm^3 or $\boxed{3030 \text{ cm}^3}$

NOTE: on your exam paper, DO NOT write 3.03E+3, but 3.03 x 10^3 or simply 3030 cm^3
The "E" for scientific notation is a calculator notation ONLY.

A closed box (like a pizza box) is to be formed from a sheet of cardboard 64 cm by 40 cm by cutting equal squares, of side x cm, from two corners of the short side, and two equal rectangles of width x cm from the other two corners and folding along the dotted lines as shown in the diagram.





Find the volume of the box as a function of x and hence find the maximum volume.

$$\text{Volume} = x(40-2x)\left(\frac{64-2x}{2}\right)$$

simplify

$$= x(40-2x)(32-x)$$

Quartic Polynomials

a) Sketch
b) Local max. and min.
c) x-intercepts
d) y-intercepts.
e) domain and range
Note all of them domain is R.
But Range depends on the sketch.