Laws of Probability

Probability 4

Chapter 8, Pages 354 – 363

Combined Events | Mutually Exclusive Events | Independent Events | Conditional Probability | IB Questions

Combined Events

- The probability of two (A & / or B) events occurring (A ∪ B) is given by the addition law of probability.

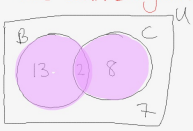
booklet

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Worked Examples

1. In a class of 30 students, 15 own a bicycle, 10 own a car and 2 own both. Find the probability that a randomly chosen student owns either a bicycle or a car.

Use Venn Diagram



$\frac{13+2+8}{30} = \frac{23}{30}$

Use Addition Law

$P(B) = \frac{15}{30}$

$P(C) = \frac{10}{30}$

$P(B \cap C) = \frac{2}{30}$

$P(B \cup C) = ?$

$P(B \cup C) = \frac{15}{30} + \frac{10}{30} - \frac{2}{30} = \frac{23}{30}$

Mutually Exclusive Events

- A and B are mutually exclusive events if they can't occur together (eg heads & tails).
- If A and B are mutually exclusive events then $P(A \cap B) = 0$, so the probability of A or B occurring is given by:

$$P(A \cup B) = P(A) + P(B)$$

Worked Examples

2. Furcan is an excellent marathon athlete. The probability that he will come in first place in the school marathon is 0.56. The probability that he comes in second place is 0.38. Calculate the probability that Furcan finishes the marathon in either first or second place.

$P(1st) = 0.56$

$P(2nd) = 0.38$

P(1 U 2)

mutually exclusive because he can't be both 1st & 2nd

$P(1st \text{ or } 2nd) = 0.56 + 0.38$

$= 0.94$

Independent Events

- Independent events occur when both events can occur together, but the occurrence of one doesn't effect the occurrence of the other.
- The probability of both occurring together is:

$$P(A \cap B) = P(A) \times P(B)$$

in booklet

in tree diagrams along a branch

Worked Examples

3. The probability I will be late for school on any one day is 0.05. The probability that I will eat rice for lunch is 0.23. Given that the two events are independent, find
   a) the probability that I am late for school and eat rice for lunch
   b) the probability that I am late for school or eat rice for lunch

$P(L) = 0.05 \quad P(R) = 0.23$

a) $P(L \cap R) = 0.05 \times 0.23 = 0.0115$

b) $P(L \cup R) = P(L) + P(R) - P(L \cap R)$
$= 0.05 + 0.23 - 0.0115$
$= 0.2685$

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Conditional Probability

- Conditional probability is when the probability of one event happening depends on another event which has previously occured.
- P(A | B) means "the probability of event A occurring given that event B has occurred.

clue = conditional prob.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ * booklet

event which has occured

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Worked Examples

4. The probability that Janie passes a mathematics test is 0.73. The probability that she passes a French test is 0.65. The probability that she passes neither is 0.08. She has a mathematics and French test on the same day. Her teacher told her she has passed the French test. What is the probability that she also passed the mathematics test?

what is the prob she passed both F & M given that she passed F?

$P(M) = 0.73 \quad P(F) = 0.65 \quad P(M \cup F)' = 0.08$

$P(M \cup F) = 1 - 0.08 = 0.92$ = prob of passing math or French or both.

Need to find $(M \cap F)$ ← can't assume they are independent

$P(M \cup F) = pM + pF - p(M \cap F)$

$0.92 = 0.73 + 0.65 - p(M \cap F) \quad p(M \cap F) = 1.38 - 0.92 = 0.46$

$p(M|F) = p\frac{M \cap F}{pF} = \frac{0.46}{0.65} = 0.708$

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Worked Examples

5. In a class of 30 students, 12 like to play golf, 17 like to play hockey and 3 play neither game.
   Find a) the probability that a student chosen at random likes to play golf.
   b) the probability that a student chosen at random likes to play golf and hockey.
   c) the probability that a student chosen at random likes to play hockey given that the student likes to play golf.

use laws and = ∩

$P(G) = \frac{12}{30} \quad P(H) = \frac{17}{30} \quad P(G \cup H)' = \frac{3}{30}$

a) $\frac{12}{30}$

b) $p(G \cap H)$: $P(G \cup H) = pG + pH - p(G \cap H)$

$\frac{27}{30} = \frac{12}{30} + \frac{17}{30} - p(G \cap H)$

$p(G \cap H) = \frac{29}{30} - \frac{27}{30} = \frac{2}{30}$

c) $P(H|G) = \frac{pG \cap H}{pG} = \frac{2}{30} \div \frac{12}{30} = \frac{2}{30} \times \frac{30}{12} = \frac{2}{12}$

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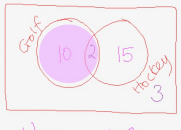
Worked Examples

5. In a class of 30 students, 12 like to play golf, 17 like to play hockey and 3 play neither game.
   Find a) the probability that a student chosen at random likes to play golf.
   b) the probability that a student chosen at random likes to play golf and hockey.
   c) the probability that a student chosen at random likes to play hockey given that the student likes to play golf.

same Q Venn dia



$30 - 3 = 27$

$12 + 17 = 29$

$29 - 27 = 2$ ↑ intersection

a) $\frac{12}{30}$ b) $\frac{2}{30}$ c) $\frac{2}{12}$

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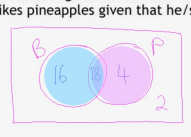
Worked Examples

6. In a class of 40, 34 like bananas, 22 like pineapples and 2 dislike both fruits.
   a) Draw a Venn diagram to represent this situation.
   If a student is randomly selected, find the probability that the student:
   b) likes both fruits
   c) likes at least one fruit BUP
   d) likes bananas given that he/she likes pineapples
   e) dislikes pineapples given that he/she likes bananas.



$40 - 2 = 38$

$34 + 22 = 56$

$56 - 38 = (18)$ intersection

b) $\frac{18}{40}$ c) $\frac{38}{40}$ d) $\frac{18}{22}$

e) $\frac{16}{34}$

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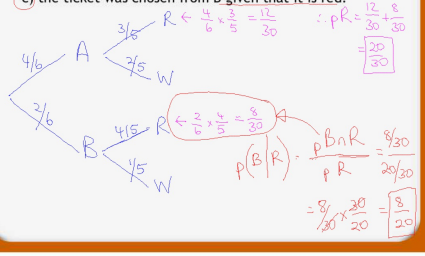
Worked Examples

7. Bin A contains 3 red and 2 white tickets. Bin B contains 4 red and 1 white. A die with 4 faces marked A and 2 faces marked B is rolled and used to select bin A or B. A ticket is then selected from this bin.
   a) Draw a tree diagram to show this information.
   Determine the probability that:
   b) the ticket is red
   c) the ticket was chosen from B given that it is red.



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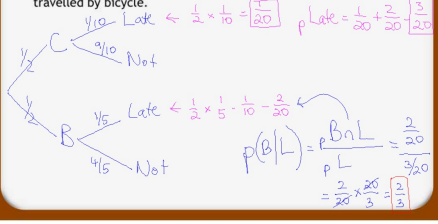
Worked Examples

9. Amos travels to school either by car or by bicycle. The probability of being late for school is 1/10 if he travels by car and 1/5 if he travels by bicycle. On any particular day he is equally likely to travel by car or by bicycle.
(a) Draw a probability tree diagram to illustrate this information.
(b) Find the probability that
   (i) Amos will travel by car and be late.
   (ii) Amos will be late for school.
(c) Given that Amos is late for school, find the probability that he travelled by bicycle.



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Homework

Ex 8K, p358: Q1, 3, 5
Ex 8L, p361: Q1, 3, 5
Ex 8M, p363: Q1, 3, 5

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