Quadratic Functions: Graphically

Homework:
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What do you do with expressions?

- **Simplify them**
- **Factor them**

What do you do with equations/inequalities?

- **Solve them**

What do you do with functions?

- **From an equation, create a table.**
- **From a table, write the equation.**
- **From an equation, graph the curve (lines are just straight curves)**
- **From a graph, write an equation**
- **Give Domain and Range, regardeless of how the function is given.**
  - **mapping diagram**
  - **odered pairs**
  - **graph**
  - **equation**
  - **table**
- **Find x & y intercepts (if any)**
- **Given the equation or graph of the function,**
  - **a. Evaluate the function for a given value of x**
  - **b. Find the value of x that makes the function an specific value.**
  - **c. Check to see if a point is <u>on</u> the curve.**
  - **d. If one coordinate of a point is given, find the other one.**

    (if you have the x, you can get the y, and if you have the y, you canget the x)

With quadratic functions we........

Like in all functions!

- Domain and Range:

Example: A parabola that goes on for ever: "No stop signs"

❑ Domain: *All Real Numbers*

❑ Range: *If happy parabola, (min, ∞)* / *If unhappy parabola, (-∞,max)*

- x & y-intercepts.

y-intercepts: make x=0
x-intercetps: make y=0
Universal Concept!

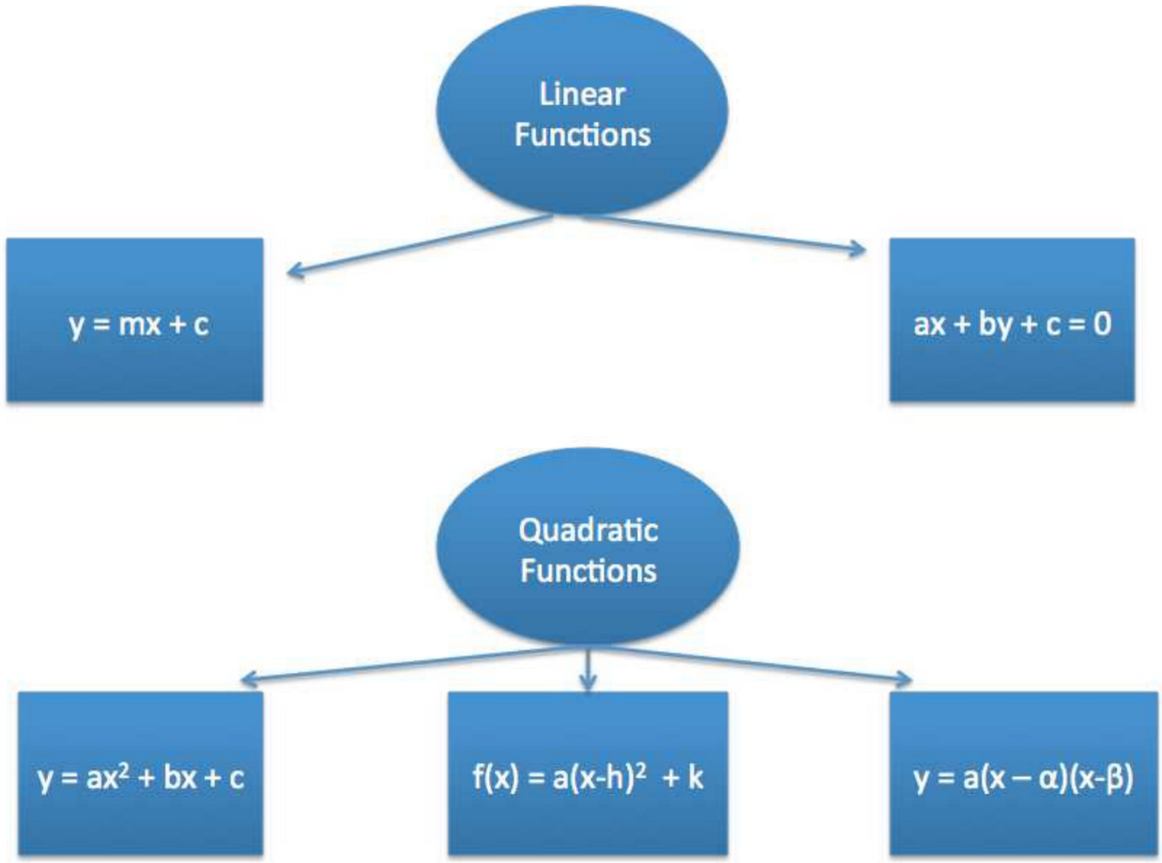
- From the equation draw the graph.
- From a graph, write the equation.
- A mapping
- A table
- Find the value of f(x) for a given x. Same as saying find the value of the function for x=2

  ex: for find f(2) $f(x) = x^2 + 2x - 3$

  $f(2) = (2)^2 + 2(2) - 3 = 5$

- If f(x) =12 what is x?

  $12 = x^2 + 2x - 3$ Now we solve for x



Standard Form

- Is it a polynomial function?

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} ... + d$$

- Standard FORM:

$$f(x) = ax^2 + bx + c$$

Understand the graph

$$ax^2 + bx + c = f(x) \text{ where } a \neq 0$$

or = y

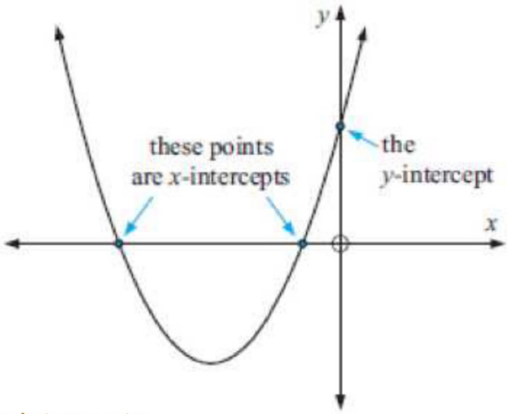
❑ y-intercept:

If the quadratic function is given in standard form. The y-intercept is always the "c" term.

This makes sense, because if you make x=0 (which is true for ALL y-intercepts) then .....

$$y = a(0)^2 + b(0) + c$$
$$y = 0 + 0 + c$$
$$y = c$$

Hence, y-int. (0, c)



❑ x-intercepts

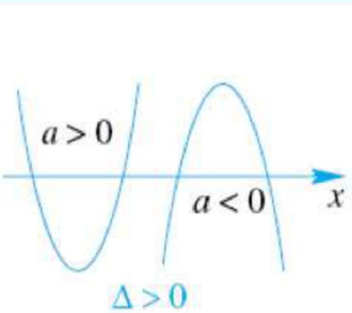
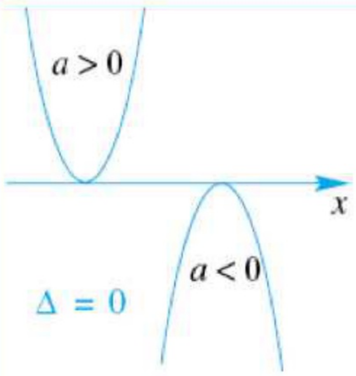
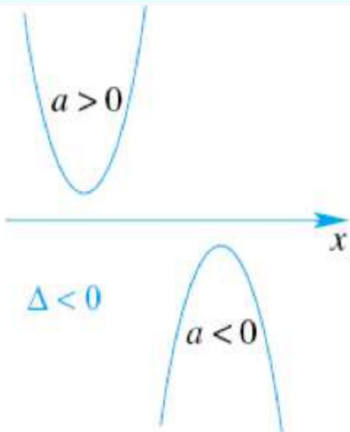
Here, it is trickier. NOT ALL parabolas have x-intercepts.
You can check next slide for a quick check to see if parabola has x-intercepts or not.

To find the x-intercepts (some parabolas have only ONE x-intercept which happens to be the vertex) you must make y=0, and solve the equation for x.

$$0 = ax^2 + bx + c$$

The Discriminant: Graphically

- The *discriminant* of a quadratic ($\Delta = b^2 - 4ac$) tells you how many **solutions** the function has. Graphically, this value represents the number of ZEROS or ROOTS.

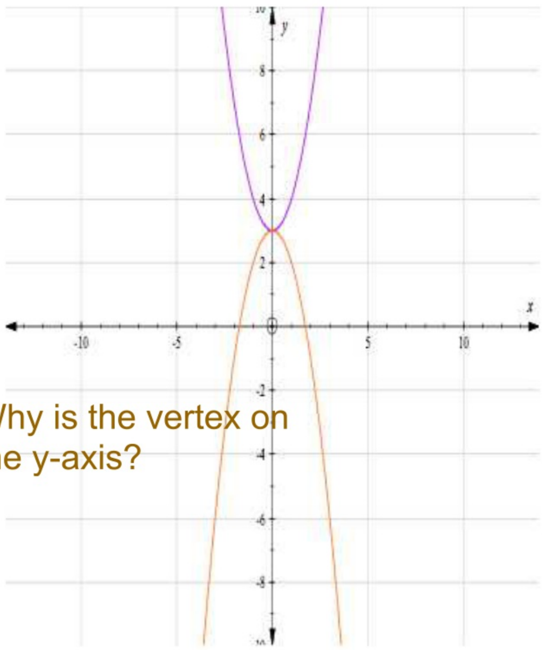
If $\Delta = b^2 - 4ac > 0$, then there are **two x–intercepts**.	If $\Delta = b^2 - 4ac = 0$, then there is **one x–intercept**.	If $\Delta = b^2 - 4ac < 0$, then there are **no x–intercepts**.
		

Observe and compare these parabolas

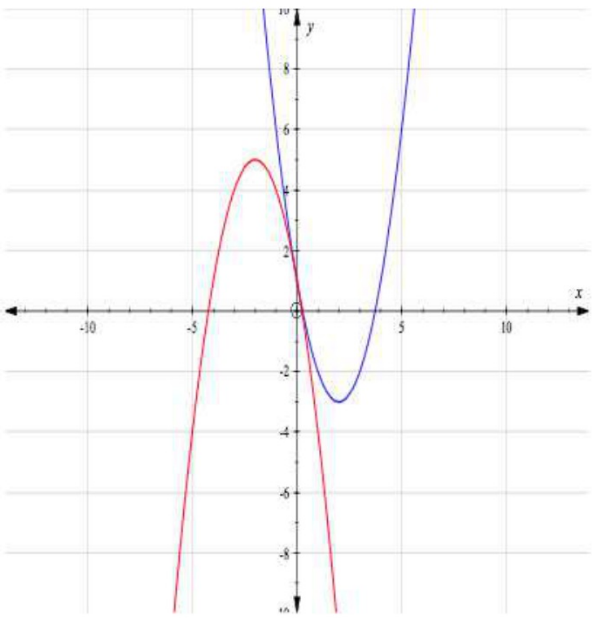
$y = -x^2 + 3$ and $y = x^2 + 3$

$y = -x^2 - 4x + 1$ and $y = x^2 - 4x + 1$

What makes happy or unhappy?



Why is the vertex on the y-axis?

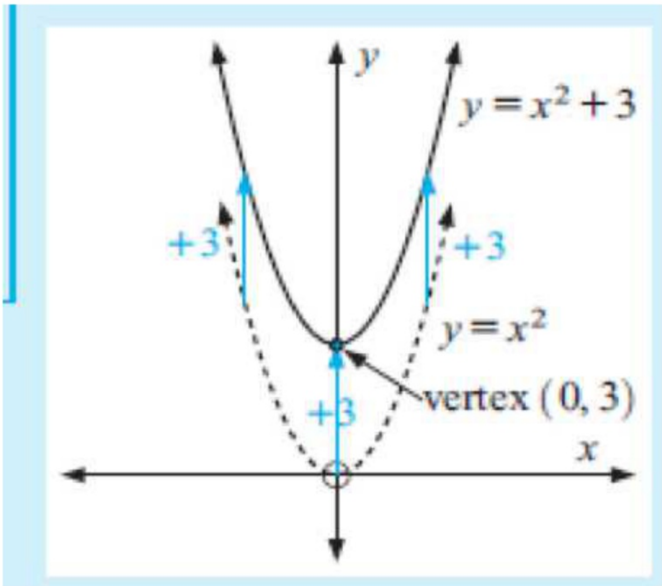


Understand the graph

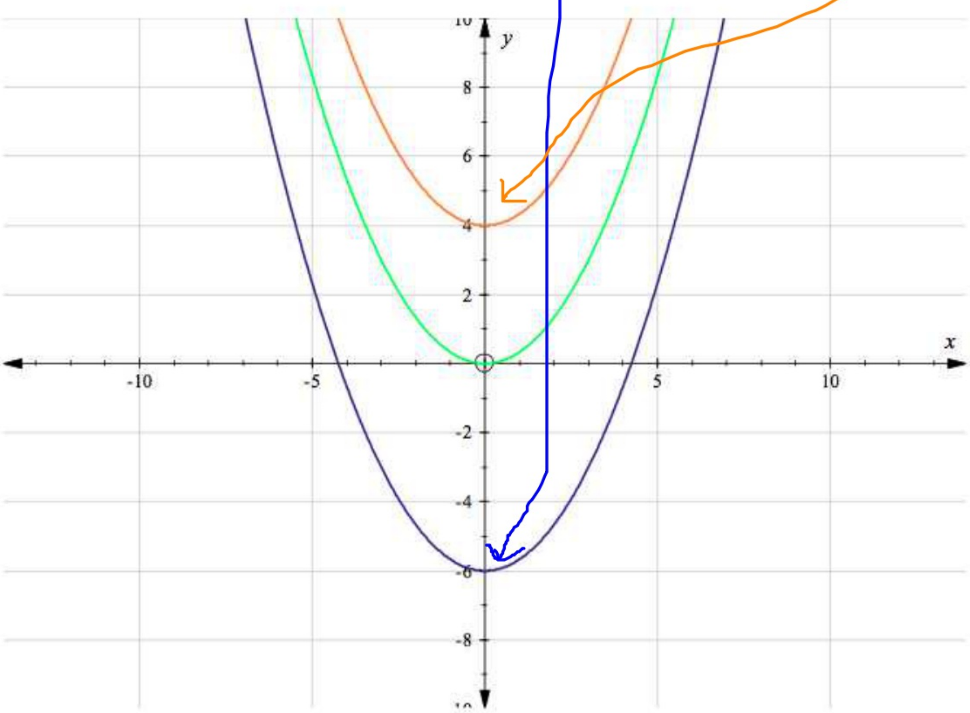
$ax^2 + bx + c = \text{f(x)}$ where $a \neq 0$ and $b = 0$

$$\therefore ax^2 + c = 0$$

Because b=0 (no x-term) the y-intercept happens to be the vertex.



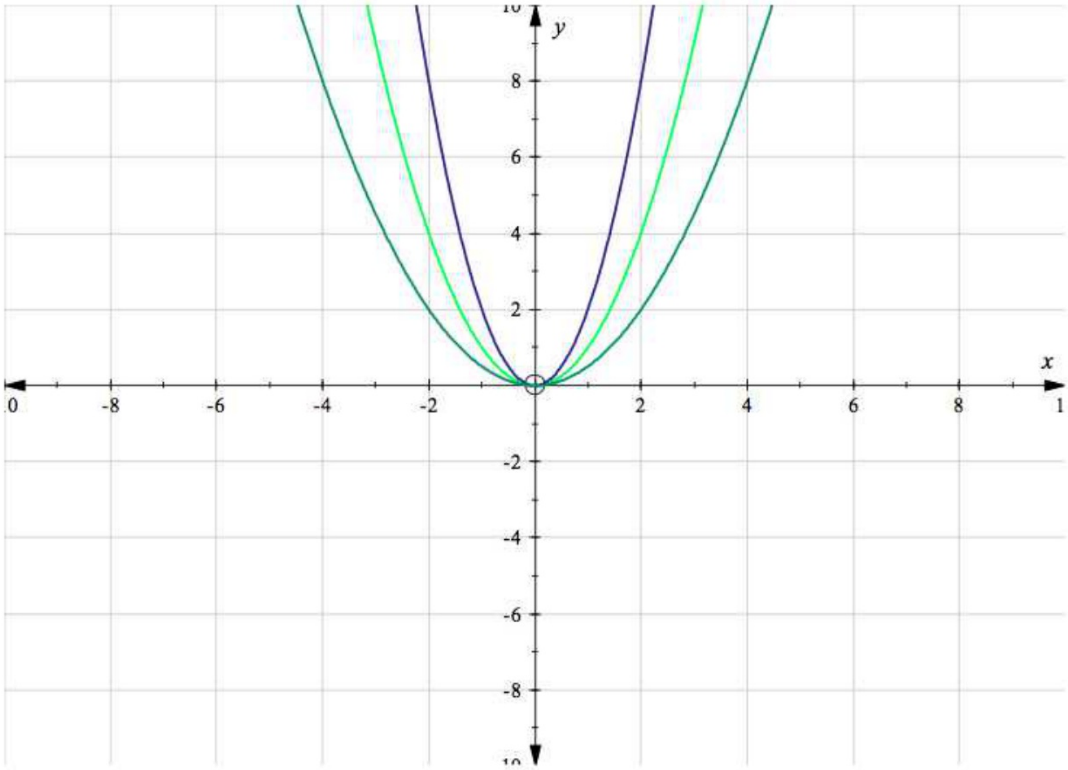
a) $y = \frac{1}{3}x^2$; b) $y = \frac{1}{3}x^2 - 6$; c) $y = \frac{1}{3}x^2 + 4$



Fatter

Thinner

a) $y = x^2$; b) $y = \frac{1}{2}x^2$; c) $y = 2x^2$



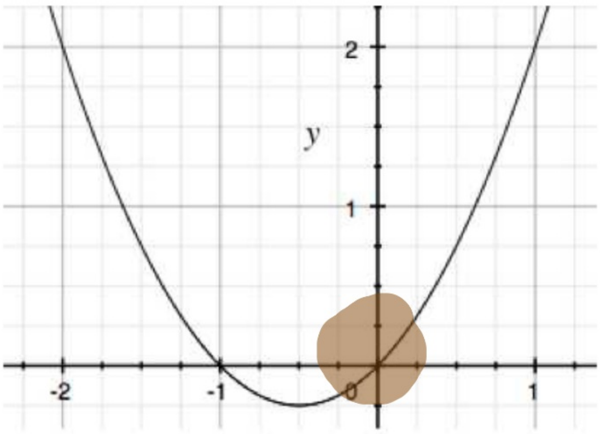
Understand the graph

$ax + bx = f(x)$ where $a \neq 0, b \neq 0, \& c = 0$

Because c=0, one of the x-intercepts is ALWAYS the origin (0 , 0)

- Graph

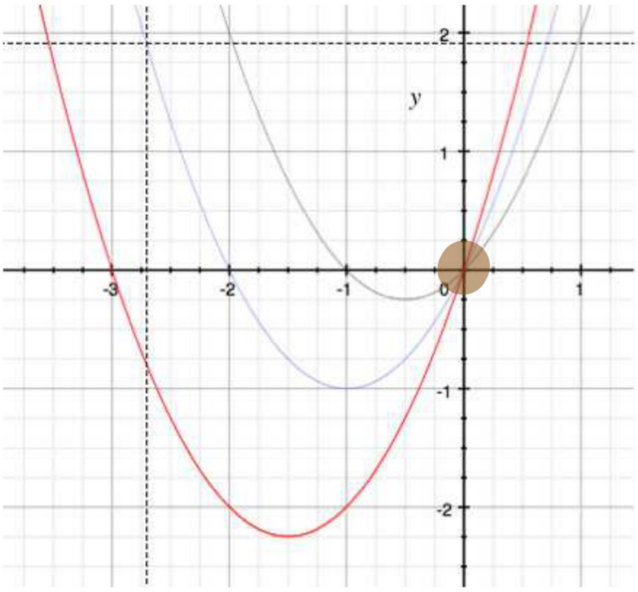
$$f(x) = x^2 + x$$

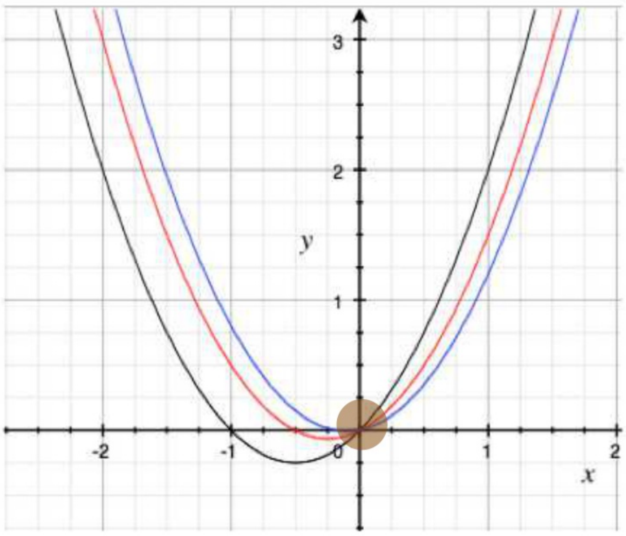


Chaging the "b", changes the shape!

$$y = x^2 + x; \quad y = x^2 + 2x; \quad y = x^2 + 3x$$

$$y = x^2 + x; \quad y = x^2 + \frac{1}{2}x; \quad y = x^2 + \frac{1}{5}x$$





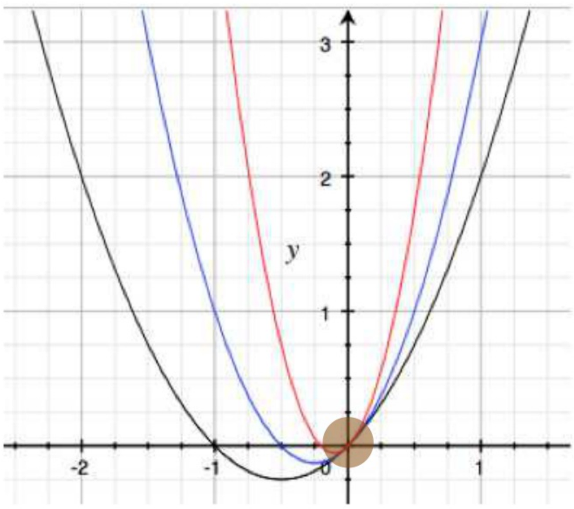
Changing the "a", changes the shape (fatter/thinner)!

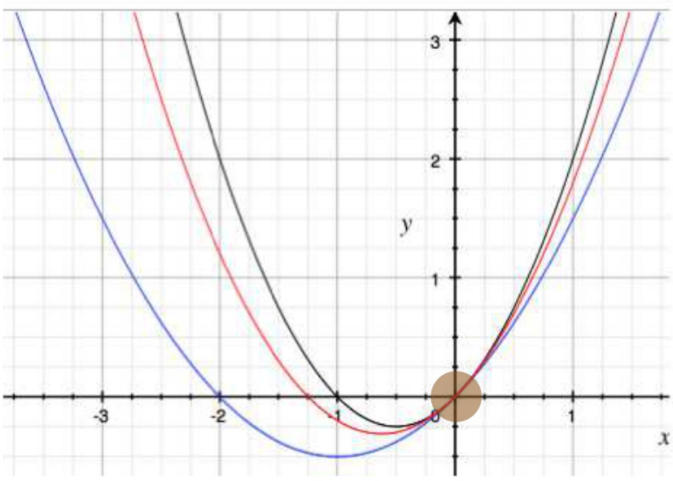
Perfect shape! Thinner Even thinnier!

$$y = x^2 + x; \quad y = 2x^2 + x; \quad y = 5x^2 + x$$

Perfect shape! Fatter Fatter, but less.

$$y = x^2 + x; \quad y = \frac{1}{2}x^2 + x; \quad y = \frac{4}{5}x^2 + x$$





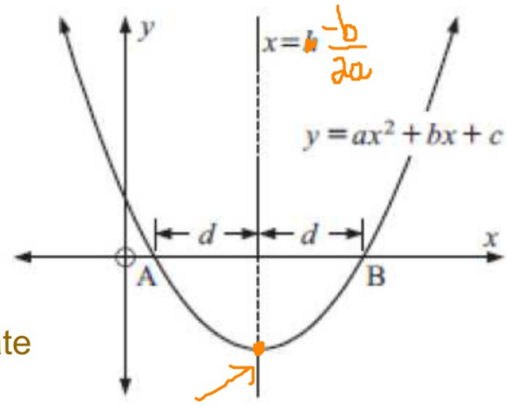
Understand the graph $ax^2+bx+c=f(x)$ where $a\neq 0$

❑ Axis of symmetry

Always write the equation of AoS.

x = "a number" after all it is a Vertical Line.

$$x=\frac{-b}{2a}$$



❑ Vertex (turning point)

It should be obvious to you, that the x-coordinate of the vertex should be -b/2a

Vertex always sits on the AoS.

$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Solve for the y-coordinate of the vertex, by substituting the x-coord.

Understand the graph $ax^2+bx+c=f(x)$ where $a\neq 0$

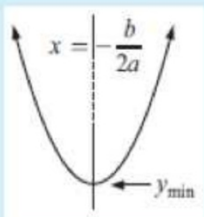
❑ Maximum & minimum points

If happy, then you have a MINIMUM

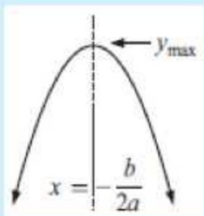
If unhappy, then you have a MAXIMUM.

For $y=ax^2+bx+c$:

- if $a>0$, the minimum value of y occurs at $x=-\frac{b}{2a}$



- if $a<0$, the maximum value of y occurs at $x=-\frac{b}{2a}$.



❑ Domain and Range

Domain:

$\{x : x\in\mathbb{R}\}$ or $\mathbb{R}$ or $(-\infty,\infty)$

Range:

for happy parabolas:

$\{y \mid y\geq \text{minimum } (y_v)\}$ or $[\text{min}, \infty)$

For unhappy parabolas:

$\{y : y\leq \text{maximum } (y_v)\}$ or $(-\infty, \text{max}]$

Note: as explained ealier (domain & range presentation) vertices are always "included"

Now let's graph the parabola!

$$ax^2 + bx + c = f(x) \text{ where } a \neq 0$$

$a=1 \quad b=-7 \quad c=15$

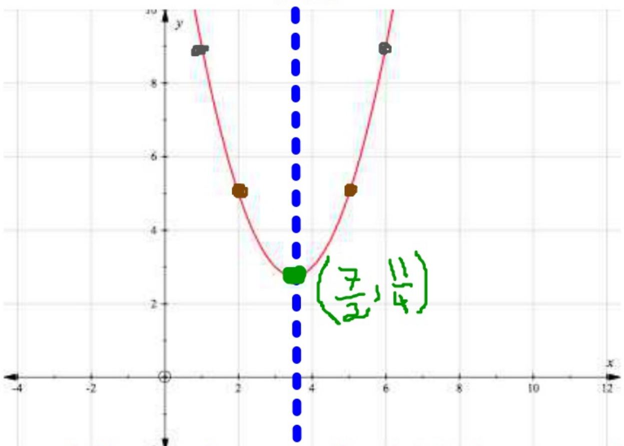
- Let's graph $y = x^2 - 7x + 15$

1. UP ☺ or down ☹
2. Does it have x-intercepts? note: you do not need to do this first. It is just an extra.
3. Get the axis of symmetry.
4. Vertex
5. Make a table of SMART points.

1. AoS $x = \frac{-(-7)}{2(1)} = 7/2$

Now, you must draw this imaginary line.

x=7/2



$\left(\frac{7}{2}, \frac{11}{4}\right)$

2. find vertex:
y= $(7/2)^2$ - 7(7/2) +15
y= 11/4 or 2.75
Now, you must plot it.

3. Table of smart points ("husband & wifes")

x	y
2	5
5	5
1	9
6	9

4. Now you have enough points to draw a smooth parabola.