Quadratic 2-Factor Form & Vertex Form

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Factor Form of Quadratic Function

- Factor Form: General
  $f(x) = a(x - p)(x - q)$
- y-intercept *As always x=0*
  $f(0)$ or $y = a(0 - p)(0 - q)$
- Axis of Symmetry
  $\frac{p+q}{2}$
- Vertex
  $V\left(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right)\right)$
- Direction of Opening
  - a > 0
  - a < 0
  - What if a = 0?
    It wouldn't be a quadratic function!!!

- Example: $f(x)$ or $y = 2(x-3)(x+2)$
  So here, p = 3 and q = -2, can you see it?
  Follow the formula!!!
- y-intercept: $y = 2(0 - 3)(0 + 2)$
  $f(0) = -12 \quad (0, -12)$
- Axis of Symmetry
  $\frac{3-2}{2} \Rightarrow x = \frac{1}{2}$
- Vertex
  $V\left(\frac{1}{2}, -\frac{25}{2}\right)$ or $V(0.5, -12.5)$
- Direction of Opening

Can you understand why the AoS is the midpoint between the two intercepts?

$$y=2(x-3)(x+2)$$

From this form we know that the x-intercepts are (3,0) and (-2,0), but do you know why?

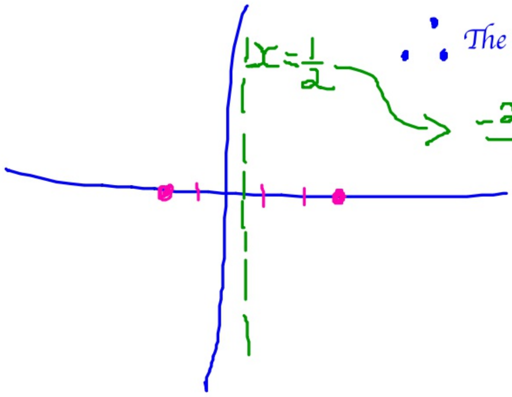
Look what happens when I substitute 0 for y, and solve for x!

$$0=2(x-3)(x+2)$$

$$x-3=0 \quad \text{or} \quad x+2=0$$

$$x=3 \qquad\qquad x=-2$$

$\therefore$ The x-intercepts have to be (3,0) and (-2,0)



$$\frac{-2+3}{2}=\frac{1}{2}$$

Even if you are very good at manipulating quadratics in the standard form, DO NOT spend the time to rearrange the quadratic function given in factor form into standard form!!!
IT IS A WASTE OF YOUR TIME.

$$y=2(x-3)(x+2)$$

$$2(x^2+2x-3x-6)$$

$$2(x^2-x-6)$$

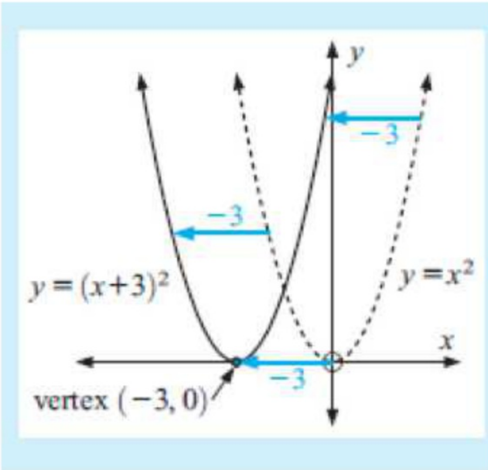
$$y=2x^2-2x-12$$

Same parabola!

Examples

$y=a(x-p)(x-q)$

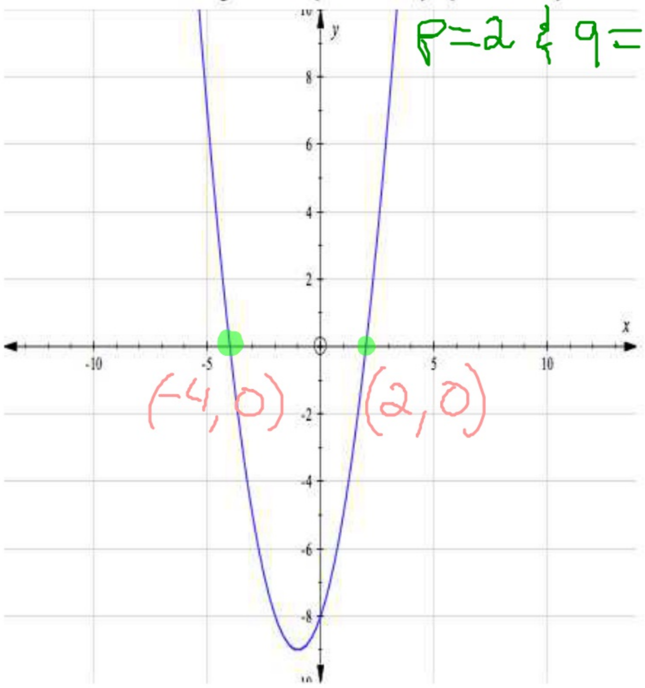
$y=(x+3)^2$ *or* $(x+3)(x+3)$



$-(-4)$

$y=(x-2)(x+4)$

$p=2$ & $q=-4$



Match the given graphs to the possible formulae stated:

a $y=2(x-1)(x-4)$ C

b $y=-(x+1)(x-4)$ E

c $y=(x-1)(x-4)$ B

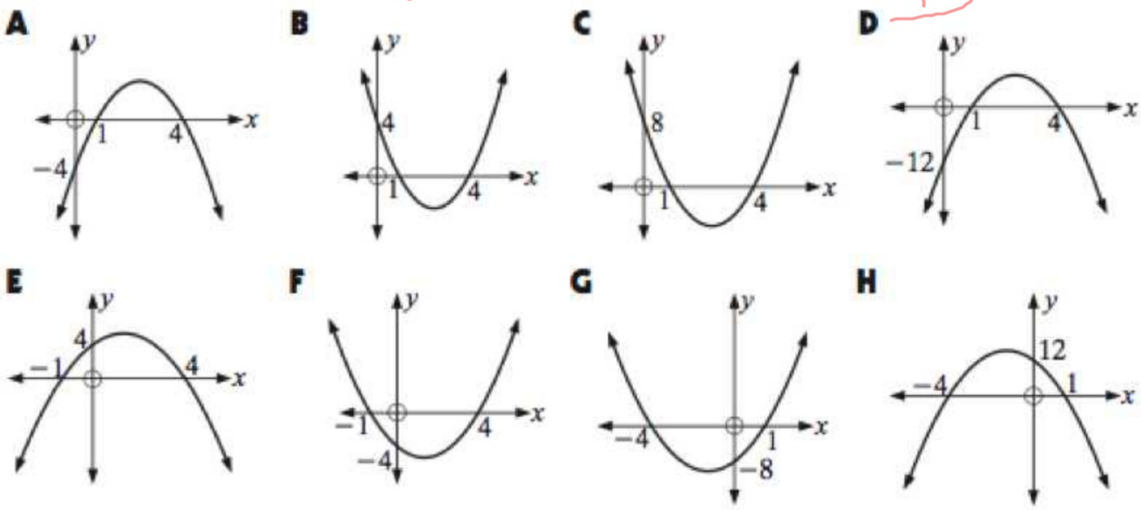
d $y=(x+1)(x-4)$ F

e $y=2(x+4)(x-1)$ G

f $y=-3(x+4)(x-1)$ H

g $y=-(x-1)(x-4)$ A

h $y=-3(x-1)(x-4)$ D



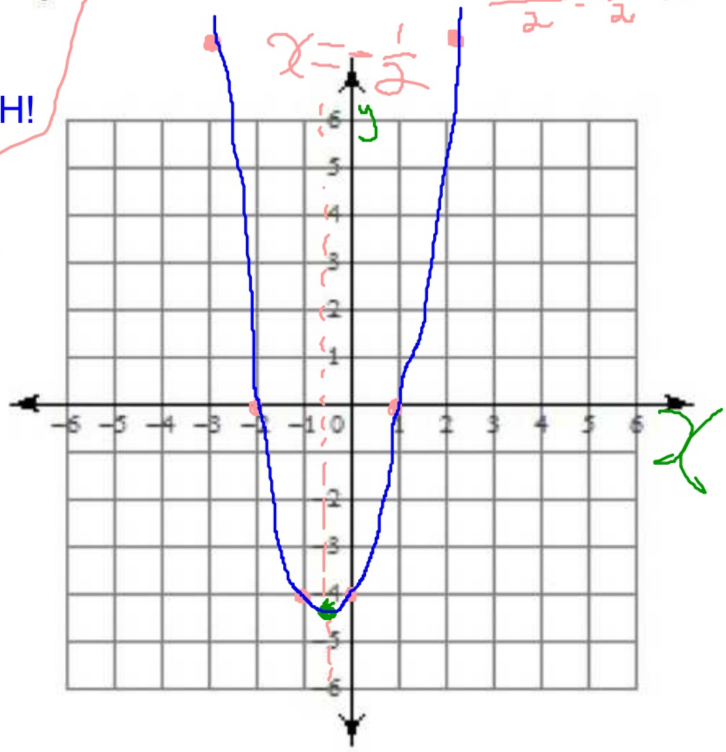
Graph the function using the factor form: Do you need to change it to standard form?

- Let's graph y= 2(x-1)(x+2)

Use GDC or midpoint formula $\frac{p+q}{2}$ $\frac{-2+1}{2} = -\frac{1}{2}$

1. UP ☺ or down ☹
2. Does it have x-intercepts? DUH!
   (-2,0) & (1,0)
3. Get the axis of symmetry. $x = -\frac{1}{2}$
4. Vertex $\left(-\frac{1}{2}, -\frac{9}{2}\right)$
5. Make a table of "smart" points

x	y
-2	0
1	0
-1	-4
0	-4
-3	8
2	8



Vertex Form of a Quadratic Function

- Vertex Form: General
  $f(x)$ or $y = a(x - h)^2 + k$
- y-intercept: As always, $x = 0$
  y or $f(0) = a(0 - h)^2 + k$
- Vertex is at:
  $V(h, k)$
- Axis of Symmetry
  $x = h$
- Direction of Opening
  - a > 0
  - a < 0
  - What if a = 0?

Then it is not a quadratic function!
But linear.

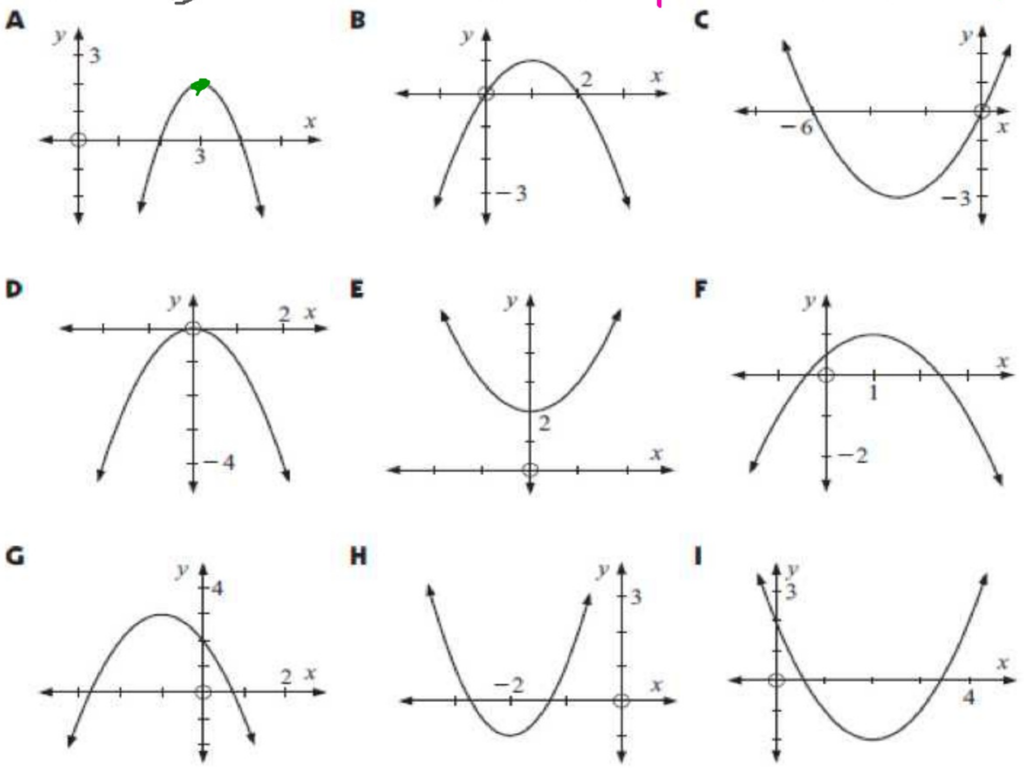
- Example: $y = 2(x - 3)^2 - 4$
  Hence $h = 3$ and $k = -4$
- y-intercept:
  y or $f(0) = 2(0-3)^2 - 4$ (0,14)
  $= 14$
- Vertex is at:
  $V(3, -4)$
- Axis of Symmetry
  $x = 3$
- Direction of Opening

Match each quadratic function with its corresponding graph:

a $y = -1(x+1)^2 + 3$ G **b** $y = -2(x-3)^2 + 2$ A **c** $y = x^2 + 2$ E

d $y = -1(x-1)^2 + 1$ B **e** $y = (x-2)^2 - 2$ I **f** $y = \frac{1}{3}(x+3)^2 - 3$ C

g $y = -x^2$ D **h** $y = -\frac{1}{2}(x-1)^2 + 1$ F **i** $y = 2(x+2)^2 - 1$ H



Graph the function from Vertex Form!
Are you going to transfer it to Standard Form?

- Let's graph $y = -2(x-4)^2 - 1$

1. UP ☺ or down ☹
2. Does it have x-intercepts?
3. Plot the Vertex $V(4,-1)$
4. Axis of Symmetry $x = 4$
5. Make a table of "smart" points

note: who cares at this point

You can use your GDC!!

x	y
3	-3
5	-3
2	-9
6	-9