**Extra Review of Sequences & Series**

**1.** Consider the arithmetic sequence 1, 4, 7, 10, 13, …

(a) Find the value of the eleventh term.

(2)

(b) The sum of the first *n* terms of this sequence is (3*n* – 1).

(i) Find the sum of the first 100 terms in this arithmetic sequence.

(ii) The sum of the first *n* terms is 477.

(a) Show that 3*n*2– *n* – 954 = 0.

(b) Using your graphic display calculator or otherwise, find the number of terms, *n.*

(6)

(Total 8 marks)

**2.** A concert choir is arranged, per row, according to an arithmetic sequence. There are 20 singers in the fourth row and 32 singers in the eighth row.

(a) Find the common difference of this arithmetic sequence.

(3)

There are 10 rows in the choir and 11 singers in the first row.

(b) Find the **total** number of singers in the choir.

(3)

(Total 6 marks)

**3.** An arithmetic sequence is defined as

*un* = 135 + 7*n*, *n* = 1, 2, 3, …

(a) Calculate *u*1, the first term in the sequence.

(2)

(b) Show that the common difference is 7.

(2)

*Sn* is the sum of the first *n* terms of the sequence.

(c) Find an expression for *Sn.* Give your answer in the form *Sn* = *An*2+ *Bn*, where *A* and *B* are constants.

(3)

The first term, *v*1, of a geometric sequence is 20 and its fourth term *v*4 is 67.5.

(d) Show that the common ratio, *r*, of the geometric sequence is 1.5.

(2)

*Tn* is the sum of the first *n* terms of the geometric sequence.

(e) Calculate *T*7, the sum of the first seven terms of the geometric sequence.

(2)

(f) Use your graphic display calculator to find the smallest value of *n* for which *Tn* > *Sn*.

(2)

(Total 13 marks)

**4.** (a) common difference = 3 (may be implied) (A1)  
*u*11 = 31 (A1)(G2)

(b) (i) (3 × 100 – 1) **OR ** (M1)14950 (A1)(G2)

(ii) (a) (3*n* – 1) = 477 **OR **(2 + 3(*n* – 1)) = 477 (M1)  
3*n*2 *– n* = 954 (M1)  
3*n*2 *– n* *–* 954 = 0 (AG)

**Notes:** Award second (M1)for correct removal of denominator **or** brackets and no further incorrect working seen.  
Award at most (M1)(M0)if last line not seen.

(b) 18 (G2)

**Note:** If both solutions to the quadratic equation are seen and the correct value is not identified as the required answer, award (G1)(G0).

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**5.** (a) 20 **=** *u*1 *+* 3*d* (A1)32 **=** *u*1 *+* 7*d* (A1)

**Note:** Award (A1) for each equation, (A1) for correct answer.

**OR**

*d* =  (A1)(A1)

**Note:** Award (A1) for numerator, (A1) for denominator.

*d* **=** 3 (A1) (C3)

(b) (2 × 11 **+** 9 × 3) ***or*** (11 **+** 38) (M1)(A1)(ft)

**Note:** Award (M1) for correct substituted formula, (A1) for correct substitution, follow through from their answer to part (a).

**OR**

11 **+** 14 **+** ... **+** 38 (M1)(A1)(ft)

**Note:** Award (M1) for attempt at the sum of a list, (A1)(ft) for all correct numbers, follow through from their answer to part (a).

= 245 (A1)(ft) (C3)

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**6.** (a) *u*1 = 135 + 7(1) (M1)  
 = 142 (A1)(G2)

(b) *u*2 = 135 + 7(2) = 149 (M1)  
*d* = 149 – 142 **OR** *alternatives* (M1)(ft)  
*d* = 7 (AG)

(c) *Sn* =  (M1)(ft)

**Note:** Award (M1) for correct substitution in correct formula.

=  **OR** *equivalent* (A1)  
=  (= 3.5*n*2 + 138.5*n*) (A1)(G3)

(d) 20*r*3 = 67.5 (M1)  
*r*3 = 3.375 **OR** *r* =  (A1)  
*r* = 1.5 (AG)

(e) *T*7 =  (M1)

**Note:** Award (M1) for correct substitution in correct formula.

= 643 *(accept* *643.4375)* (A1)(G2)

(f)  (M1)

**Note:** Award (M1) for an attempt using lists or for relevant graph.

*n* = 10 (A1)(ft)(G2)

**Note:** Follow through from their (c).

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**7.** Consider the geometric sequence 16, 8, *a*, 2, *b*, …

(a) Write down the common ratio.

(1)

(b) Write down the value of

(i) *a*;

(ii) *b*.

(2)

(c) The sum of the first *n* terms is 31.9375. Find the value of *n*.

(3)

(Total 6 marks)

**8.** The population of big cats in Africa is increasing at a rate of 5 % per year. At the beginning of 2004 the population was 10 000.

(a) Write down the population of big cats at the beginning of 2005.

(1)

(b) Find the population of big cats at the beginning of 2010.

(2)

(c) Find the number of years, from the beginning of 2004, it will take the population of big cats to exceed 50 000.

(3)

(Total 6 marks)

**9.** The first term of an arithmetic sequence is 0 and the common difference is 12.

(a) Find the value of the 96th term of the sequence.

(2)

The first term of a geometric sequence is 6. The 6th term of the geometric sequence is equal to the 17th term of the arithmetic sequence given above.

(b) Write down an equation using this information.

(2)

(c) Calculate the common ratio of the geometric sequence.

(2)

(Total 6 marks)

**10.** Consider the geometric sequence 8, *a*, 2,… for which the common ratio is .

(a) Find the value of *a*.

(b) Find the value of the eighth term.

(c) Find the sum of the first twelve terms.

(Total 6 marks)

**11.** Two students Ann and Ben play a game. Each time Ann passes GO she receives $15. Each time Ben passes GO he receives 8% of the amount he already has. Both students start with $100.

(a) How much money will Ann have after she has passed GO 10 times?

(b) How much money will Ben have after he passes GO 10 times?

(c) How many times will the students have to pass GO for Ben to have more money than Ann?

(Total 6 marks)

**12.** (a) 0.5  (A1) (C1)

(b) (i) 4 (A1)

(ii) 1 (A1) (C2)

(c)  = 31.9375 (M1)(M1)

**Note:** Award (M1) for correct substitution in the GP formula, (M1) for equating their sum to 31.9375

**OR**

sketch of the function *y* =  (M1)

indication of point where *y* **=** 31.9375 (M1)

**OR**

16 + 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125 + 0.0625 = 31.9375 (M1)(M1)

**Note:** Award (M1) for a list of at least 7 correct terms, (A1) for the sum of the terms equated to 31.9375.

*n* **=** 9 (A1)(ft) (C3)

**Note:** Follow through from their answer to part (a) but answer mark is lost if n is not a whole number.

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**13.** (a) 10000 × 1.05  
= 10 500 (A1) (C1)

(b) 10000 × 1.056 (M1)

**Note:** Award (M1) for correct substitution into correct formula.

= 13400 (A1) (C2)

(c) 50 000 = 10000 × 1.05*n* (M1)(A1)

**Note:** Award (M1) for 10000 × 1.05n or equivalent, (A1) for = 50000

*n* = 33.0 (Accept 33) (A1) (C3)

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**14.** (a) *u*96 = *u*1 + 95*d* (M1)

= 0 + 95 ×12

= 1140 (A1) (C2)

(b) 6*r*5 = 16*d* (A1)

6*r*5= 16 × 12 (192) (A1)

**Note:** (A1) only, if both terms seen without an equation. (C2)

(c) *r*5 = 32 (A1)(ft)

**Note:** (ft) from their (b)

*r* = 2 (A1)(ft) (C2)

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**15.** (a) 

*a* = 4 (A1)

**OR**



*a* = 4 (A1) (C1)

(b)  (M1)(A1)(ft)

**OR**

 (M1)(A1)(ft) (C2)

(c)  (M1)(A1)(ft) (A1)(ft) (C3)

**Note:** Award (M1) for using correct formula and correct substitution, (A1) for correct answer (15.99...). (A1) for correct answer to 3 s.f.

[6]

**16.** (a) 100 + 15 × 10 (M1)  
= 250 (A1)  
**OR**250 (using table function of the GDC) (G2) (C2)

(b) 100(1.08)10 (M1)  
= 215.89 (A1)  
**OR**  
215.89 (using table function of the GDC) (G2) (C2)

(c) 100 + 15*x* = 100(1.08)*x* (M1)  
After 16 years (A1)

**Note:** Candidate can use trial and error so not  
necessary to see the first line to award (A2).

**OR**16 years (using table function of the GDC). (G2) (C2)

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