Introduction to Probability

Probability 2

Chapter 8,
Pages 352 – 355 & 364 - 368

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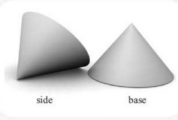
Experimental Probability

- Number of trials is the total number of times the experiment is repeated.
- Outcomes are the different results possible for each trial.
- The frequency of an outcome is the number of times it occurred.
- relative frequency is the frequency expressed as a fraction or percentage of the total number of trials. = estimated experimental probability

Worked Examples

1. A small plastic cone was tossed into the air 279 times.
It fell on its side 183 times and on its base 96 times.



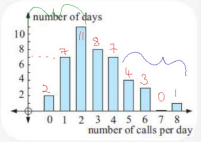
a) How many trials were made? 279
b) What are the outcomes for this experiment? side base
c) What is the frequency of the cone landing on its base? 96
d) What is the relative frequency for each outcome? base = 96/279 side = 183/279 exptal prob.

NOTE: in examples such as this where probability can only be determined by experiment,
experimental probability = relative frequency

The more trials conducted, the more accurate the estimate of probability.

Worked Examples

2. Betul keeps records of the number of phone calls she receives over a period of consecutive days.



a) How many days did her survey last? 43
b) Estimate Betul's chance of receiving:
i) no phone calls on one day 2/43
ii) 5 or more phone calls on one day 8/43
iii) less than 3 phone calls on a day. 2+7+11=20 20/43

advice - always use fractions and usually don't simplify

Sample Space

- A sample space is the set of all possible outcomes of an experiment.
- Depending on the complexity of outcomes, there are 3 common ways to represent the sample space.

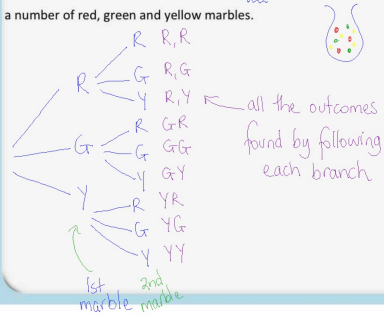
1. List all possible outcomes
eg flipping a coin, the sample space = heads, tails
2. Grid for when there are two events happening at once.
eg flipping two coins, the sample space is

1 coin \ coin 2	H	T
H	•	•
T	•	•

← 4 outcomes

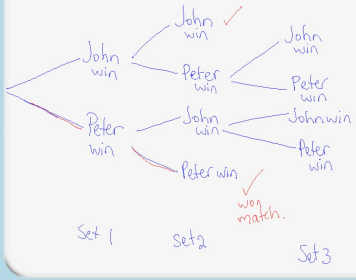
Sample Space

3. Tree diagrams can represent complex events, sequential events
eg, show the sample space for drawing 2 marbles from a bag containing a number of red, green and yellow marbles.



Worked Examples

4. John plays Peter at tennis and the first to win two sets wins the match. Make a tree diagram to show all possible outcomes.



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Theoretical Probability

- In many situations the probability of an outcome can be predicted based on what we theoretically would expect to occur.
- Eg, a normal ("fair", unbiased) 6-sided dice has an equal probability of landing on any side, so we don't need lots of trials to know that the probability of throwing a one = $\frac{1}{6}$
- P(event) = $\frac{\text{number of members of the event } E}{\text{total number of possible outcomes}}$
- Note: the probability for any event is between 0 and 1 (impossible, certain)

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Worked Examples

5. A ticket is randomly selected from a basket containing 3 green, 4 yellow and 5 blue tickets. Determine the probability of getting:
   a) a green ticket
   b) a green or yellow ticket
   c) an orange ticket
   d) a green, yellow or blue ticket.

a) $\frac{3}{12} = \frac{1}{4}$

b) $\frac{7}{12}$

c) $\frac{0}{12} = 0$

d) $\frac{12}{12} = 1$

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Complementary Events

- Complementary events are those where one of them must occur. – either or the other
- Eg. on a dice; P(2) = $\frac{1}{6}$; P(not 2) = $\frac{5}{6}$
- Getting a 2 and not getting a 2 are complementary events
- If E is an event, then E' is the complement of E

$P_E + P_{E'} = 1$

Or, $P(E \text{ not occuring}) = 1 - P(E \text{ occuring})$

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Worked Examples

6. A marble is randomly selected from a box containing 5 green, 3 red and 7 blue marbles. Determine the probability that the marble is:
   a) red
   b) not red
   c) green
   d) neither green nor blue
   e) green or red

a) $\frac{3}{15} = \frac{1}{5}$

b) $\frac{12}{15} = \frac{4}{5}$

c) $\frac{5}{15} = \frac{1}{3}$

d) $\frac{3}{15} = \frac{1}{5}$

e) $\frac{8}{15}$

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Worked Examples

7. List the 8 possible 3-child families, according to the gender of the children. Eg, GGB means *"the first is a girl, the second is a girl and the third is a boy"*.

GGG BBB
GGB BBG
GBG BGB
GBB BGG

a) Assuming each of these is equally likely to occur, determine the probability that a randomly selected 3-child family consists of:

i) all boys $\frac{1}{8}$

ii) boy, girl, then girl $\frac{1}{8}$

iii) two girls and a boy $\frac{3}{8}$

iv) an eldest girl $\frac{1}{2}$

v) at least one boy. $\frac{7}{8}$ $(1 - P(\text{no boys}))$ use complementary. $= 1 - \frac{1}{8} = \frac{7}{8}$

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Worked Examples

The table below shows the number of left and right handed tennis players in a sample of 50 males and females.

If a tennis player was selected at random from the group, find the probability that the player is

(a) male and left handed;

(b) right handed;

(c) right handed, given that the player selected is female.

	Left handed	Right handed	Total
Male	3	29	32
Female	2	16	18
Total	5	45	50

a) $\frac{3}{50}$

b) $\frac{45}{50}$

c) $\frac{16}{18}$

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Homework

Ex 8I Q1, 3, 5, 7 * homework

Ex 8J, p354: Q2, 4 and 6

Ex 8N, p365: Q1, 3 and 4

Sunday

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