

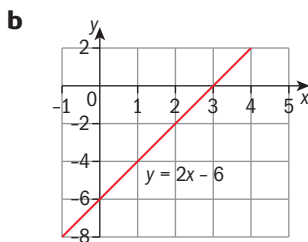
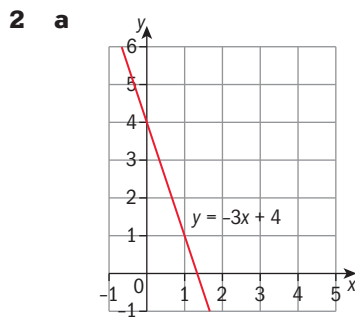
# 5

# Statistical applications

## Answers

### Skills check

- 1 a** mean = 3.61 standard deviation = 1.21  
The small standard deviation implies that the data are close to the mean
- b** mean = 14 standard deviation = 0.643  
The mean is the middle data value (14) since the frequencies are symmetrical about this value. The standard deviation is very small since most of the data values equal the mean and the rest are close to it



### Exercise 5A

- 1 a**
- 
- b** 0.16    **c** 0.815    **d**  $200 \times 0.16 = 32$
- 2 a**
- 
- b** 81.5%    **c**  $100 \times 0.5 = 50$

- 3 a**
- 
- b** 16%    **c**  $60 \times 0.68 = 40.8$  or 41
- 4 a**
- 
- b** 0.025    **c**  $75 \times 0.84 = 63$

### Exercise 5B

- 1 a**
- 
- b** 0.0766  
**c**  $365 \times 0.0766 = 27.959$  or 28 days
- 2 a i**  $P(\text{IQ} < 90) = 0.159$   
**ii**  $P(\text{IQ} > 120) = 0.0228$   
**iii**  $P(80 < \text{IQ} < 110) = 0.819$
- b**  $P(\text{IQ} > 115) = 0.0668$ ,  $2000 \times 0.0668 = 134$
- 3 a** 0.0668    **b** 0.00621    **c**  $300 \times 0.927 = 278$
- 4 a**
- 
- b** 0.0401  
**c**  $80 \times 0.0122 = 0.976$  or 1
- 5 a** 78.9%  
**b** 0.00621  
**c**  $100 \times 0.0304 = 3.04$  or 3
- 6** 0.106
- 7 a** 86.4%  
**b**  $30 \times 0.0304 = 0.912$  or 1

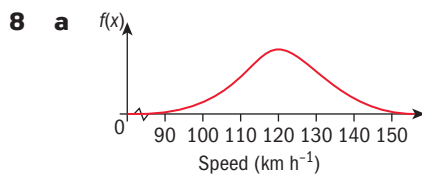
8 mean = 1.78 m standard deviation = 0.02 m

- a 0.00621
- b 3

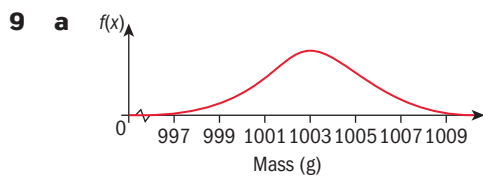
**Exercise 5C**

- 1  $p = 4.93$
- 2  $h = 183$
- 3  $k = 20.8$
- 4  $w = 222$
- 5 a 3.47 to 4.99 kg
- b  $180 \times 0.683 = 123$
- c 0.0685
- d 87.7%
- e  $w = 5.48$
- 6 a  $a = 29, b = 30, c = 31$
- b 0.919
- c  $d = 32.8$
- d  $5000 \times 0.6246 \dots$   
= 3123 (accept 3120 to 3125)

- 7 a 0.000429
- b 0.854
- c  $t = 5885$



- b 62.5%
- c  $p = 106$
- d  $800 \times 0.911 = 729$
- e  $800 \times 0.159 = 127$

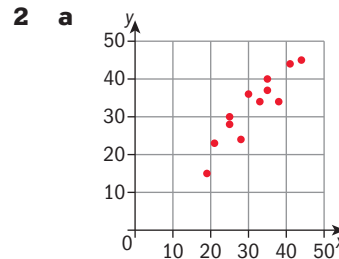


- b 0.0228      c 0.0668
- d  $400 \times 0.0668 = 26.7$  or 27
- e  $p = 1006$
- 10 a 0.466%
- b A baby weighing 2.34 kg (2.34 is nearer the mean than 5.5).
- c  $300 \times 0.0808 = 24.2$  or 24
- d  $w = 3.16$

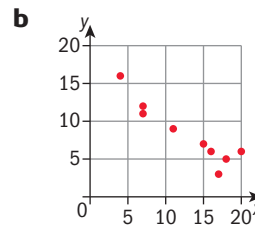
**Exercise 5D**

- 1 a strong positive linear
- b moderate negative linear
- c moderate positive linear

- d weak positive linear
- e none
- f perfect negative linear
- g non-linear
- h weak negative linear

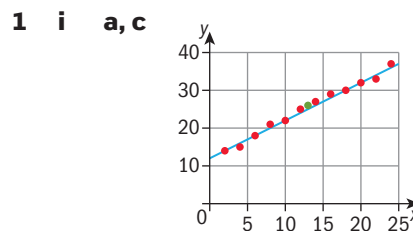


moderate positive linear correlation



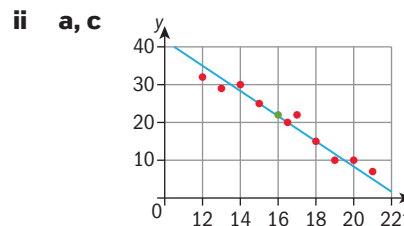
moderate negative linear correlation

**Exercise 5E**



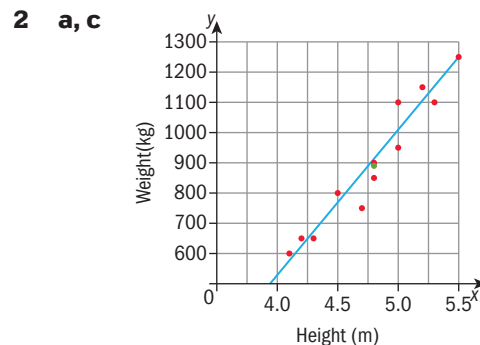
strong positive correlation

- b mean of  $x = 13$ , mean of  $y = 25.75$



strong negative correlation

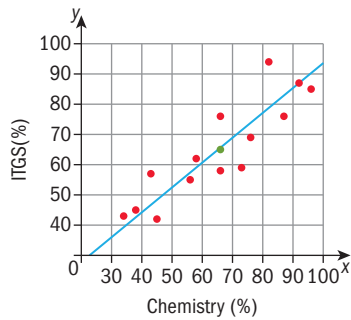
- b mean of  $x = 16.5$ , mean of  $y = 20.2$



moderate positive correlation

- b mean height = 4.78 m  
mean weight = 896 kg
- d 820 kg

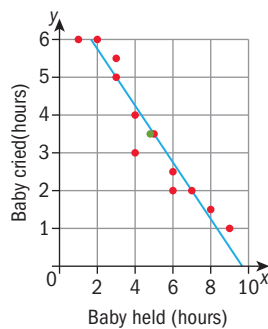
3 a, c



weak positive correlation

- b chemistry mean = 65.3  
ITGS mean = 65.1
- d 52%

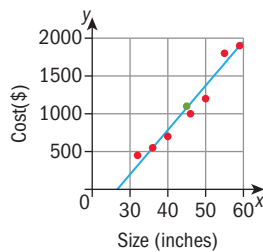
4 a, c



moderate negative correlation

- b mean number of hours held = 4.83  
mean number of hours spent crying = 3.5
- d 4.5 hours

5 a, c



moderate positive correlation

- b mean screen size = 45.6 inches  
mean cost = \$ 1100
- d \$1540

### Exercise 5F

- 1  $r = 0.931$ , strong positive correlation
- 2 a  $r = 0.880$   
b strong positive correlation
- 3  $r = -0.891$ , strong negative correlation
- 4  $r = 0.936$ , strong positive correlation
- 5  $r = 0.990$ , strong positive correlation
- 6  $r = 0.200$ , very weak positive correlation
- 7  $r = 0.985$ , strong positive correlation
- 8  $r = 0.580$ , moderate positive correlation

### Exercise 5G

- 1 a  $r = 0.994$ , strong positive correlation  
b  $y = 1.47x + 116$   
c  $y = 1.47(1000) + 116 = 1586$ , £ 1590 (3 s.f)
- 2 a  $r = 0.974$   
b  $y = 0.483x + 15.6$   
c  $y = 0.483(8) + 15.6 = 19.464$ , 19.5 cm
- 3 a  $\bar{x} = 68.6$   $s_x = 6.55$   $\bar{y} = 138$   $s_y = 5.97$   
b  $r = -0.860$   
c strong negative correlation  
d  $y = -0.784x + 192$   
e  $y = -0.784(70) + 192 = 137.12$ ,  
137 seconds
- 4 a  $r = 0.792$   
b  $y = 0.193x + 1.22$   
c  $y = 0.193(15) + 1.22 = 4.115$ , 4.12
- 5 a  $y = 0.0127x + 0.688$   
b  $y = 0.0127x(70) + 0.688 = 1.577$ , 1.58 AUD
- 6 a  $y = 0.751x + 11.6$   
b  $y = 0.751(50) + 11.6 = 49.15$ , 49 situps
- 7 a  $y = 1.04x - 2.53$   
b  $y = 1.04(60) - 2.53 = 59.87$ , 59.9
- 8 a  $y = 0.279x + 2.20$   
b  $y = 0.279(40) + 2.20 = 13.36$ , 13.4 hours.

### Exercise 5H

- 1 a  $H_0$ : Genre of book is independent of age  
 $H_1$ : Genre of book is dependent on age  
b  $\frac{97}{300} \times \frac{130}{300} \times 300 = 42.0$   
c  $(3 - 1)(3 - 1) = 4$   
d  $\chi^2_{\text{calc}} = 26.9$   
e  $26.9 > 9.488$ , therefore we reject the null hypothesis. There is enough evidence to conclude that genre of book is dependent on age. ( $p$ -value = 0.0000207 < 0.05)
- 2 a  $H_0$ : Hair color and eye color are independent  
 $H_1$ : Hair color and eye color are dependent.  
b  $\frac{85}{227} \times \frac{90}{227} \times 227 = 33.7$   
c  $(3 - 1)(3 - 1) = 4$   
d  $\chi^2_{\text{calc}} = 44.3$   
e  $44.3 > 7.779$ , therefore we reject the null hypothesis. There is enough evidence to conclude that hair colour and eye color are dependent.  
( $p$ -value = 0.00000000556 < 0.1)

- 3 a**  $H_0$ : Favorite flavor is independent of race.  
 $H_1$ : Favorite flavor is dependent on race.
- b**  $\frac{35}{140} \times \frac{44}{140} \times 140 = 11$
- c**  $(4 - 1)(3 - 1) = 6$
- d**  $\chi^2_{\text{calc}} = 0.675$
- e**  $0.675 < 12.59$ , therefore we do not reject the null hypothesis. There is enough evidence to conclude that favourite flavor is independent of race. ( $p$ -value =  $0.995 > 0.05$ )
- 4 a**  $H_0$ : Film genre is independent of gender  
 $H_1$ : Film genre is dependent on gender
- b**  $\frac{39}{80} \times \frac{21}{80} \times 30 = 10.2$
- c**  $(2 - 1)(4 - 1) = 3$
- d**  $\chi^2_{\text{calc}} = 19.0$
- e**  $19.0 > 11.345$ , therefore we reject the null hypothesis. There is enough evidence to conclude that film genre is dependent on gender. ( $p$ -value =  $0.000276 < 0.01$ )
- 5 a**  $H_0$ : Grade is independent of the number of hours  
 $H_1$ : Grade is dependent on the number of hours
- b**  $\frac{90}{220} \times \frac{96}{220} \times 220 = 39.3$
- c**  $(3 - 1)(3 - 1) = 4$
- d**  $\chi^2_{\text{calc}} = 42.1$
- e**  $42.1 > 9.488$ , therefore we reject the null hypothesis. There is enough evidence to conclude that grade is dependent on number of hours spends playing computer games. ( $p$ -value =  $0.000000159 < 0.05$ )
- 6 a**  $H_0$ : Employment grade is independent of gender  
 $H_1$ : Employment grade is dependent on gender
- b**
- |        | Directors | Management | Teachers |
|--------|-----------|------------|----------|
| Male   | 11.5      | 71.5       | 538.9    |
| Female | 20.5      | 127.5      | 960.1    |
- c**  $(2 - 1)(3 - 1) = 2$
- d**  $\chi^2_{\text{calc}} = 180$
- e**  $180 > 4.605$ , therefore we reject the null hypothesis. There is enough evidence to conclude that employment grade is dependent on grade. ( $p$ -value =  $8.08 \times 10^{-40} < 0.1$ )
- 7 a**  $H_0$ : Amount of sushi sold is independent the day of the week  
 $H_1$ : Amount of sushi sold is dependent on the day of the week.
- b**  $\frac{70}{470} \times \frac{145}{470} \times 470 = 52.4$
- c**  $(3 - 1)(3 - 1) = 4$
- d**  $\chi^2_{\text{calc}} = 0.840$
- e**  $0.840 < 9.488$ , therefore we do not reject the null hypothesis. There is enough evidence to conclude that the amount of sushi sold is independent of the day of the week. ( $p$ -value  $0.933 > 0.05$ )
- 8 a**  $H_0$ : A puppy's weight is independent of its parent's weight.  
 $H_1$ : A puppy's weight is dependent on its parent's weight
- b**  $\frac{46}{141} \times \frac{41}{141} \times 141 = 13.4$
- c**  $(3 - 1)(3 - 1) = 4$
- d**  $\chi^2_{\text{calc}} = 13.$
- e**  $13.7 > 13.277$ , therefore we reject the null hypothesis. There is enough evidence to conclude that a puppy's weight is dependent on its parent's weight.
- 9 a**  $H_0$ : Music preference is independent of age  
 $H_1$ : Music preference is dependent on age
- b**  $\frac{137}{419} \times \frac{101}{419} \times 419 = 33.0$
- c**  $(3 - 1)(4 - 1) = 6$
- d**  $\chi^2_{\text{calc}} = 31.5$
- e**  $31.5 > 12.59$ , therefore we reject the null hypothesis. There is enough evidence to conclude that music preference is dependent on age. ( $p$ -value =  $0.0000204 < 0.05$ )
- 10 a**  $H_0$ : Age at which a baby is potty trained is independent of gender.  
 $H_1$ : Age at which a baby is potty trained is dependent on gender.
- b**  $\frac{140}{300} \times \frac{69}{300} \times 300 = 32.2$
- c**  $(2 - 1)(3 - 1) = 2$
- d**  $\chi^2_{\text{calc}} = 51.6$
- e**  $51.6 > 4.605$ , therefore we reject the null hypothesis. There is enough evidence to conclude that the age at which a baby in potty trained is dependent on gender. ( $p = 6.23 \times 10^{-12} < 0.1$ ).

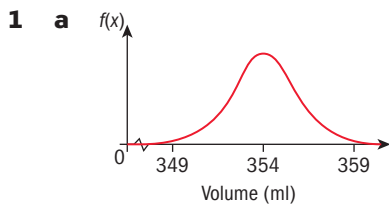
- 11 a  $H_0$ : Grade is independent of gender  
 $H_1$ : Grade is dependent on gender

	5,6 or 7	3 or 4	1 or 2
Male	25.2	66.6	15.1
Female	24.8	65.4	14.9

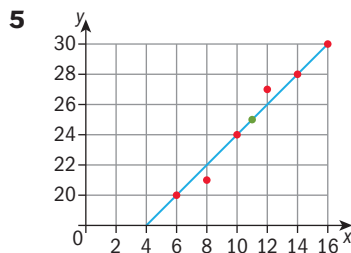
- c  $(2 - 1)(3 - 1) = 2$   
d  $\chi^2_{\text{calc}} = 0.467$   
e  $0.467 < 5.991$ , therefore we do not reject the null hypothesis. There is enough evidence to conclude that grade is independent of gender. ( $p = 0.792 > 0.05$ )

Review exercise

Paper 1 style questions



- b 0.0548      c  $100 \times 0.0548 = 5.48$ , 5 cans  
2 a 32.2%  
b  $6000 \times 0.00982 = 58.9$ , 59 people  
3 a 93.3%      b  $p = 1.01$   
4 a strong positive correlation  
b no correlation  
c moderate negative correlation



- a strong positive correlation  
b  $\bar{x} = 11$       c  $\bar{y} = 25$   
d 23  
6 a  $r = 0.980$ , strong positive correlation  
b  $y = 0.801x - 77.4$   
c  $y = 0.801(170) - 77.4 = 58.77$ , 58.8 cm  
7 a  $r = 0.810$ , strong positive correlation  
b  $y = 0.215x + 14.3$   
c  $y = 0.215x(40) + 14.3 = 22.9$  seconds

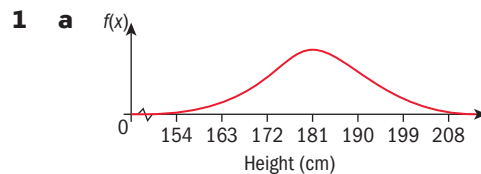
- 8  $H_0$ : Flavor of ice creams is independent of age  
 $H_1$ : Flavor of ice creams is dependent on age  
Expected values

	$x < 25$	$25 \leq x < 45$	$x \geq 45$
Vanilla	14.06	11.84	11.1
Strawberry	10.64	8.96	8.4
Chocolate	13.3	11.2	10.5

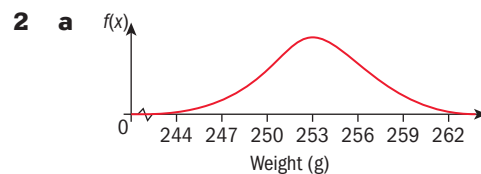
degrees of freedom =  $(3 - 1)(3 - 1) = 4$   
 $p$ -value =  $0.963 > 0.05$ ,  $\chi^2_{\text{calc}} = 0.604$   
We do not reject the null hypothesis. There is enough evidence to conclude that flavor of ice cream is independent of age.  
(critical value = 9.488, ( $\chi^2_{\text{calc}} = 0.604 < 9.488$ ))

- 9 a  $H_0$ : The number of pins knocked down is independent of which hand is used.  
b  $(2 - 1)(3 - 1) = 2$   
c  $\frac{20}{120} \times \frac{60}{120} \times 120 = 10$   
d  $p$ -value =  $0.422 > 0.1$  (significance value). Therefore we do not reject the null hypothesis. There is enough evidence to conclude that the number of pins knocked down is independent of which hand is used.  
10 a  $H_0$ : The outcome is independent of the time spent preparing for a test.  
b  $(3 - 1)(2 - 1) = 2$   
c  $p$ -value =  $0.069 > 0.05$ , therefore we do not reject the null hypothesis. There is enough evidence to conclude that the outcome is independent of the time spent preparing for a test.

Paper 2 style questions

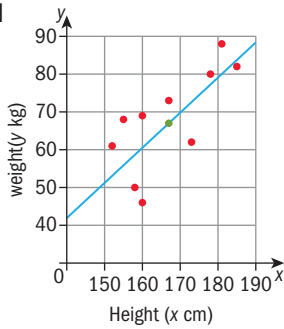


- b 0.252  
c 0.731  
d  $60 \times 0.0599 = 3.59$ , 4 men  
e  $k = 166$



- b 0.159  
c  $300 \times 0.252 = 75.6$ , 76 sweets

3 a, d



- b  $\bar{x} = 166.9$   
 c  $\bar{y} = 67.3$   
 d i  $y = 0.719x - 52.8$   
 e 69 kg
- 4 a  $r = 0.823$   
 b strong positive correlation  
 c  $y = 0.219x + 3.85$   
 d  $y = 0.29(35) + 3.85 = 11.515$   
 $= 12$  hours (nearest hr)
- 5 a  $r = 0.866 = 0.9$  (1 d.p.)  
 b strong positive correlation  
 c  $y = 0.0666x - 2.36$
- 6 a  $r = 0.887 = 0.89 = 0.89$  (2 d.p.)  
 b strong positive correlation  
 c  $y = 0.015x + 0.229$   
 d  $y = 0.0151(80) + 0.229 = 1.437, 1.44$  euros
- 7 a  $y = 0.163x - 15.0$   
 b  $y = 0.163(170) - 15.0 = 12.71$ , dress size 13  
 c  $r = 0.741$   
 d moderate positive correlation
- 8  $H_0$ : Choice of game is independent of gender  
 $H_1$ : Choice of game depends on gender  
 Expected values:

	Badminton	Table tennis	Darts
Male	39.4	14.8	26.8
Female	29.6	11.2	20.2

degrees of freedom  $= (2 - 1)(3 - 1) = 2$

$\chi^2_{\text{calc}} = 0.667$   $p$ -value  $= 0.717 > 0.05$

We do not reject the null hypothesis. There is enough evidence to conclude that choice of game is independent of gender.

(critical value  $= 5.991, \chi^2_{\text{calc}} = 0.667 < 5.991$ )

- 9 a  $p = 21.6$   $q = 14.4$   $r = 13.6$   
 b i  $H_0$ : The extra-curricular activity is independent of gender  
 ii  $(2 - 1)(3 - 1) = 2$   
 c  $\chi^2_{\text{calc}} = 4.613$   
 d  $4.613 > 4.605$ , therefore we reject the null hypothesis. There is enough evidence to conclude that extra-curricular activity is dependent on gender.
- 10 a i  $\frac{300}{500} \times \frac{180}{500} \times 500 = 108$   
 ii  $b = 12$   $c = 132$   $d = 88$   
 b  $H_0$ : position in upper management is independent of gender  
 $H_1$ : position in upper management is dependent on gender  
 c i  $\chi^2_{\text{calc}} = 54.9$   
 ii  $(2 - 1)(3 - 1) = 2$   
 iii  $54.9 > 5.991$ , therefore we reject the null hypothesis. There is enough evidence to conclude that position in upper management is dependent on gender.
- 11 a  $H_1$ : The choice of candidate is dependent on where the voter lives.  
 b  $\frac{3720}{8000} \times \frac{3680}{8000} \times 8000 = 1711$   
 c i  $\chi^2_{\text{calc}} = 58.4$   
 ii  $(3 - 1)(2 - 1) = 2$   
 d i The choice of candidate is dependent on gender.  
 ii  $58.4 > 9.21$ , therefore we reject the null hypothesis.
- 12 a  $\frac{90}{200} \times \frac{110}{200} \times 200 = 49.5$   
 b i  $H_0$ : Grade is independent of gender  
 ii  $(2 - 1)(3 - 1) = 2$   
 iii  $\chi^2_{\text{calc}} = 0.400$   
 c  $0.400 < 5.991$ , therefore we do not reject the null hypothesis. There is enough evidence to conclude that grade is independent of gender